Farmer Carter observes that the productivity of her best milking cow, Daisy varies throughout the calendar year. She believes that two major factors contribute to the variation in the cow’s daily milk production.

**Part 1**

One factor is the stage of the cow’s pregnancy and subsequently the age of her calf. The contribution to milk production, , in litres due to this factor is given by

, where is the time in months since Jan. 1st

1. What is the maximum milk production due to this factor?

* The maximum milk production is the amplitude plus the vertical translation. Thus the maximum milk production is litres.

1. What is the period of this cycle in milk production?

* It is known that the period. In the above equation, therefore the period. It is known that is the time in months since Jan. 1st, therefore the period of this cycle in milk production is 12 months.

The second factor is the availability of good food. The contribution to milk production, , in litres due to this factor is given by

, where is the time in months since Jan. 1st

1. What is the maximum milk production due to this factor?

* The maximum milk production, as stated previously, is found by adding the amplitude and the vertical translation. Therefore the maximum milk production due to this factor is litres.

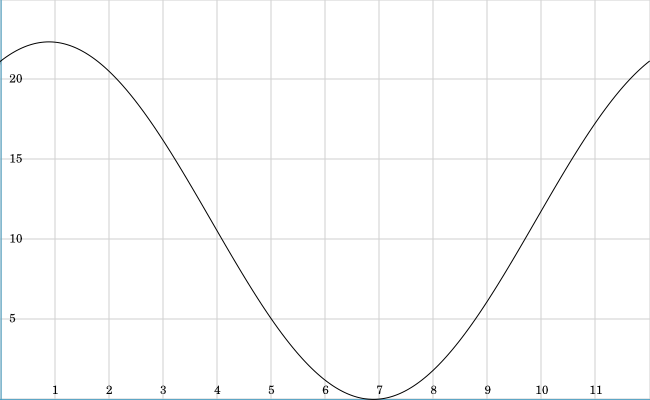
1. What is the period of this cycle in milk production?

* The period, like before is expressed as. In the above equation, , therefore the period . It is known that is the time in months since Jan. 1st, therefore the period of this cycle in milk production is 12 months.

An expression for, the cow’s actual daily milk production, in litres, is given as the sum of these two quantities.

1. Draw using technology. Sketch the graph for, labelling the maximum and minimum turning points.

**Graph of against**

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1. Use this graph to find an equation for in the form

* The amplitude of the graph is given by. Therefore the amplitude of the graph
* The vertical translation of the graph is given by. Therefore the vertical translation of the graph
* The period of the graph is the same as the highest period of the two functions. Therefore does not change, and is equal to.
* To find the value of C, we need to substitute a value of t into the original equation, and find a value for which can then be substituted into the function above.

For this, we are going to substitute the value 0 in place of t in

Substituting this point in the second equation and including all the previously found points, yields with

* Therefore, the final equation in the form is:

It should be noted that 1.0721 is an approximate value for C. For an accurate value, the equation for should be modified such that:

1. What are Daisy’s daily maximum and minimum productions and when do they occur?

* There are 31 days in January, and the maximum production occurs 0.89 months after January 1st , so the daily maximum occurs on the = 27th of January. The daily maximum production at this point is the same as the maximum on the graph, or 22.38litres.
* The daily minimum production occurs 6.89 months after January 1st. Therefore the daily minimum occurs 6 months after January 1st + 0.89 months of that month. 6 months after January 1st is July 1st. There are 31 days in July, therefore the day is the same as previously – the minimum occurs on the 27th of July, at which point Daisy only produces 0.02litres of milk.

g) If the premium daily milk production Farmer Carter expects from her cows is 20 litres, for what percentage of the year is Daisy’s production above this level.

* First, we need to find the times of the year when . Thus we can substitute into the sin equation:

Now, the distance from to is .

As this is a sin curve, , therefore is also 29 at

The second point that is when

Therefore the second point that occurs at

From this, we can conclude that the amount of months that the milk production is over 20litres is months.

Therefore the percentage of the year that the milk production is at a minimum of 20litres is .

**Part 2**

Farmer Carter has already observed that Daisy’s daily milk production was periodic and could be modelled by a function of the form

1. Show that this can be written in the form

* Using ,

1. By equating this expression for to the expression

, state a value for and and hence find the value of a to 3 decimal places.

and

Substituting the value of k into the second equation:

1. Using the identity , show that
2. Compare this result to the epression for found in Part 1

* The two equations:

Therefore, the equation found in Part 1 deviates from the equation found in Part 2 by a very small amount, due to simplification of the decimal places of the maximum and minimum values obtained from the graph of the original equation. While the equation found in Part 2 is more accurate than the one found in Part 1, it is possible to use the equation found in Part 1 as an approximate value of . When , Part 1 yields a value of 5.05, while Part 2 gives a value closer to . This means that for most values of , will result in the same month for both equations. It is suggested to use the equation found in Part 2 instead of Part 1 as it is more precise, such that it is better for daily production statistics.

**Part 3**

1. State the limitations and assumptions inherent in this model. Comment on specific features of all three functions

* It is assumed that on a year-by-year basis, the availability of good food, the stage of the cow’s pregnancy and the age of her calf remains the same as the previous year(s). This causes this model to only be accurate to 12 months. Thus it is possible that in the following year, the assumptions are changed, and the function will not be as reliable for predicting the milk production.
* For , it is assumed that the cow’s pregnancy and the age of her calf is exactly equal to what it was the previous year. If the value controlling the amplitude is changed negatively, the overall milk production per year won’t change, only there’ll be a more constant milk production rate over the months (as the maximum production gets reduced, the minimum production increases). If the value controlling the vertical translation is changed negatively, then the overall milk production per year will also be changed negatively.
* For , it is assumed that the availability of good food is exactly equal to what it was the previous year. If this value is changed negatively, then while the value for the amplitude wouldn’t affect the overall milk production per year, the vertical translation would also be changed (assumedly in the negative direction as well), thus affecting the overall milk production per year negatively.
* assumes that the values for the amplitude and vertical translation are exactly equal to what it was the previous year. Likewise in this equation, the variable controlling the vertical translation is most important, as the amplitude only controls the fluctuation of the milk production over time. If the vertical translation gets changed negatively, then the overall milk production per year is also negatively changed. It is easier for this vertical translation to be change, as it is the addition of the vertical translations of both and , thus this value is less likely to remain constant on a year-by-year basis.

1. Consider another factor that may affect milk production and use technology to develop a model to predict the resultant milk production.

* One factor which may affect the milk production may be the temperature over the year.

Using data collected [1], the averages