

1)

15.13. We note again that F_y/F is equal to the negative of the *slope* of the string at a , which is also given by $\partial y/\partial x$. Putting these together, we have

$$F_y(x, t) = -F \frac{\partial y(x, t)}{\partial x} \quad (15.20)$$

We need the negative sign because F_y is negative when the slope is positive. We write the vertical force as $F_y(x, t)$ as a reminder that its value may be different at different points along the string and at different times.

When point a moves in the y -direction, the force F_y does *work* on this point and therefore transfers energy into the part of the string to the right of a . The corresponding power P (rate of doing work) at the point a is the transverse force $F_y(x, t)$ at a times the transverse velocity $v_y(x, t) = \partial y(x, t)/\partial t$ of that point:

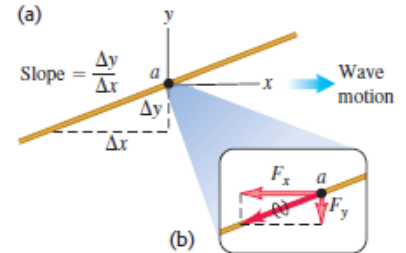
$$P(x, t) = F_y(x, t)v_y(x, t) = -F \frac{\partial y(x, t)}{\partial x} \frac{\partial y(x, t)}{\partial t} \quad (15.21)$$

This power is the *instantaneous* rate at which energy is transferred along the string. Its value depends on the position x on the string and on the time t . Note that energy is being transferred only at points where the string has a nonzero slope ($\partial y/\partial x$ is nonzero), so that there is a transverse component of the tension force, and where the string has a nonzero transverse velocity ($\partial y/\partial t$ is nonzero) so that the transverse force can do work.

Equation (15.21) is valid for *any* wave on a string, sinusoidal or not. For a sinusoidal wave with wave function given by Eq. (15.7), we have

$$\begin{aligned} y(x, t) &= A \cos(kx - \omega t) \\ \frac{\partial y(x, t)}{\partial x} &= -kA \sin(kx - \omega t) \\ \frac{\partial y(x, t)}{\partial t} &= \omega A \sin(kx - \omega t) \\ P(x, t) &= Fk\omega A^2 \sin^2(kx - \omega t) \end{aligned} \quad (15.22)$$

15.15 (a) Point a on a string carrying a wave from left to right. (b) The components of the force exerted on the part of the string to the right of point a by the part of the string to the left of point a .



$$P(x, t) = \sqrt{\mu F} \omega^2 A^2 \sin^2(kx - \omega t) \quad (15.23)$$

The \sin^2 function is never negative, so the instantaneous power in a sinusoidal wave is either positive (so that energy flows in the positive x -direction) or zero (at points where there is no energy transfer). Energy is never transferred in the direction opposite to the direction of wave propagation (Fig. 15.16).

The maximum value of the instantaneous power $P(x, t)$ occurs when the \sin^2 function has the value unity:

$$P_{\max} = \sqrt{\mu F} \omega^2 A^2 \quad (15.24)$$

To obtain the *average* power from Eq. (15.23), we note that the *average* value of the \sin^2 function, averaged over any whole number of cycles, is $\frac{1}{2}$. Hence the average power is

$$P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (\text{average power, sinusoidal wave on a string}) \quad (15.25)$$

The average power is just one-half of the maximum instantaneous power (see Fig. 15.16).

The average rate of energy transfer is proportional to the square of the amplitude and to the square of the frequency. This proportionality is a general result for mechanical waves of all types, including seismic waves (see the photo that opens this chapter). For a mechanical wave, the rate of energy transfer

2) Start with Maxwell's equations in derivative form for empty space.

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 \quad (\text{Gauss}) & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{Faraday}) \\ \nabla \cdot \mathbf{B} &= 0 \quad (\text{no name}) & \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{Ampère}) \end{aligned}$$

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\ \nabla \times (\nabla \times \mathbf{E}) &= \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) & \nabla \times (\nabla \times \mathbf{B}) &= \nabla \times \left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \\ \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} &= -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) & \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) \end{aligned}$$

Now if you look carefully, you'll see that one term in each equation equals zero and the other can be replaced with a time derivative.

$$0 - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$0 - \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial \mathbf{B}}{\partial t} \right)$$

Let's clean it up a bit and see what we get.

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E}$$

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{B}$$

3)

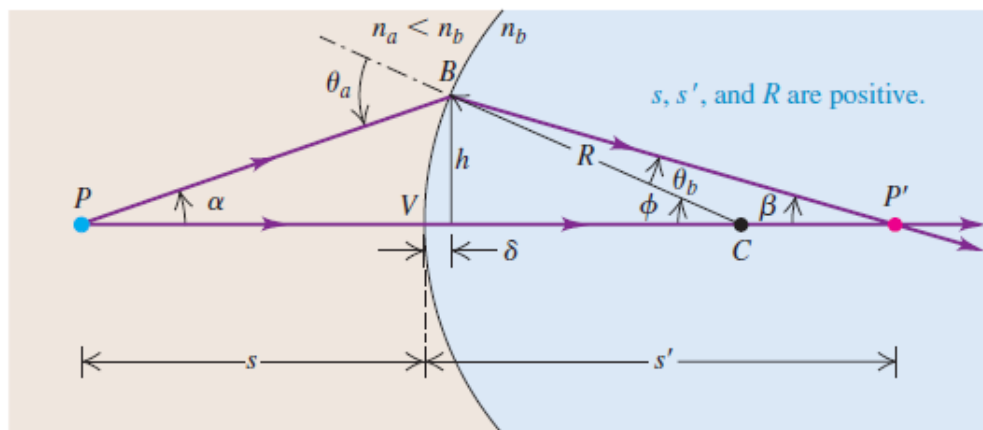


Image of a Point Object: Spherical Refracting Surface

In Fig. 34.21 a spherical surface with radius R forms an interface between two materials with different indexes of refraction n_a and n_b . The surface forms an image P' of an object point P ; we want to find how the object and image distances (s and s') are related. We will use the same sign rules that we used for spherical mirrors. The center of curvature C is on the outgoing side of the surface, so R is positive. Ray PV strikes the vertex V and is perpendicular to the surface (that is, to the plane that is tangent to the surface at the point of incidence V). It passes into the second material without deviation. Ray PB , making an angle α with the axis, is incident at an angle θ_a with the normal and is refracted at an angle θ_b . These rays intersect at P' , a distance s' to the right of the vertex. The figure is drawn for the case $n_a < n_b$. The object and image distances are both

We are going to prove that if the angle α is small, *all* rays from P intersect at the same point P' , so P' is the *real image* of P . We use much the same approach as we did for spherical mirrors in Section 34.2. We again use the theorem that an exterior angle of a triangle equals the sum of the two opposite interior angles; applying this to the triangles PBC and $P'BC$ gives

$$\theta_a = \alpha + \phi \quad \phi = \beta + \theta_b \quad (34.8)$$

From the law of refraction,

$$n_a \sin \theta_a = n_b \sin \theta_b$$

Also, the tangents of α , β , and ϕ are

$$\tan \alpha = \frac{h}{s + \delta} \quad \tan \beta = \frac{h}{s' - \delta} \quad \tan \phi = \frac{h}{R - \delta} \quad (34.9)$$

For paraxial rays, θ_a and θ_b are both small in comparison to a radian, and we may approximate both the sine and tangent of either of these angles by the angle itself (measured in radians). The law of refraction then gives

$$n_a \theta_a = n_b \theta_b$$

Combining this with the first of Eqs. (34.8), we obtain

$$\theta_b = \frac{n_a}{n_b}(\alpha + \phi)$$

When we substitute this into the second of Eqs. (34.8), we get

$$n_a \alpha + n_b \beta = (n_b - n_a) \phi \quad (34.10)$$

Now we use the approximations $\tan \alpha = \alpha$, and so on, in Eqs. (34.9) and also neglect the small distance δ ; those equations then become

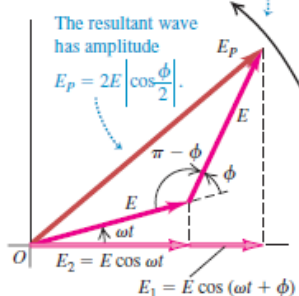
$$\alpha = \frac{h}{s} \quad \beta = \frac{h}{s'} \quad \phi = \frac{h}{R}$$

Finally, we substitute these into Eq. (34.10) and divide out the common factor h . We obtain

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad (\text{object-image relationship, spherical refracting surface}) \quad (34.11)$$

35.9 Phasor diagram for the superposition at a point P of two waves of equal amplitude E with a phase difference ϕ .

All phasors rotate counterclockwise with angular speed ω .



Amplitude in Two-Source Interference

To add the two sinusoidal functions with a phase difference, we use the same *phasor* representation that we used for simple harmonic motion (see Section 14.2) and for voltages and currents in ac circuits (see Section 31.1). We suggest that you review these sections now. Each sinusoidal function is represented by a rotating vector (phasor) whose projection on the horizontal axis at any instant represents the instantaneous value of the sinusoidal function.

In Fig. 35.9, E_1 is the horizontal component of the phasor representing the wave from source S_1 , and E_2 is the horizontal component of the phasor for the wave from S_2 . As shown in the diagram, both phasors have the same magnitude E , but E_1 is *ahead* of E_2 in phase by an angle ϕ . Both phasors rotate counterclockwise with constant angular speed ω , and the sum of the projections on the horizontal axis at any time gives the instantaneous value of the total E field at point P . Thus the amplitude E_p of the resultant sinusoidal wave at P is the magnitude of the dark red phasor in the diagram (labeled E_p); this is the *vector sum* of the other two phasors. To find E_p , we use the law of cosines and the trigonometric identity $\cos(\pi - \phi) = -\cos \phi$:

$$\begin{aligned} E_p^2 &= E^2 + E^2 - 2E^2 \cos(\pi - \phi) \\ &= E^2 + E^2 + 2E^2 \cos \phi \end{aligned}$$

Then, using the identity $1 + \cos \phi = 2 \cos^2(\phi/2)$, we obtain

$$E_P^2 = 2E^2(1 + \cos \phi) = 4E^2 \cos^2\left(\frac{\phi}{2}\right)$$

$$E_P = 2E \left| \cos \frac{\phi}{2} \right| \quad (\text{amplitude in two-source interference}) \quad (35.7)$$

You can also obtain this result without using phasors (see Problem 35.50).

When the two waves are in phase, $\phi = 0$ and $E_P = 2E$. When they are exactly a half-cycle out of phase, $\phi = \pi \text{ rad} = 180^\circ$, $\cos(\phi/2) = \cos(\pi/2) = 0$, and $E_P = 0$. Thus the superposition of two sinusoidal waves with the same frequency and amplitude but with a phase difference yields a sinusoidal wave with the same frequency and an amplitude between zero and twice the individual amplitudes, depending on the phase difference.

Intensity in Two-Source Interference

To obtain the intensity I at point P , we recall from Section 32.4 that I is equal to the average magnitude of the Poynting vector, S_{av} . For a sinusoidal wave with electric-field amplitude E_P , this is given by Eq. (32.29) with E_{max} replaced by E_P . Thus we can express the intensity in several equivalent forms:

$$I = S_{\text{av}} = \frac{E_P^2}{2\mu_0 c} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_P^2 = \frac{1}{2} \epsilon_0 c E_P^2 \quad (35.8)$$

The essential content of these expressions is that I is proportional to E_P^2 . When we substitute Eq. (35.7) into the last expression in Eq. (35.8), we get

$$I = \frac{1}{2} \epsilon_0 c E_P^2 = 2\epsilon_0 c E^2 \cos^2 \frac{\phi}{2} \quad (35.9)$$

In particular, the *maximum* intensity I_0 , which occurs at points where the phase difference is zero ($\phi = 0$), is

$$I_0 = 2\epsilon_0 c E^2$$

Note that the maximum intensity I_0 is *four times* (not twice) as great as the intensity $\frac{1}{2} \epsilon_0 c E^2$ from each individual source.

Note that the maximum intensity I_0 is *four times* (not twice) as great as the intensity $\frac{1}{2} \epsilon_0 c E^2$ from each individual source.

Substituting the expression for I_0 into Eq. (35.9), we can express the intensity I at any point very simply in terms of the maximum intensity I_0 :

$$I = I_0 \cos^2 \frac{\phi}{2} \quad (\text{intensity in two-source interference}) \quad (35.10)$$

The Lorentz Velocity Transformation

We can use Eqs. (37.21) to derive the relativistic generalization of the Galilean velocity transformation, Eq. (37.2). We consider only one-dimensional motion along the x -axis and use the term “velocity” as being short for the “ x -component of the velocity.” Suppose that in a time dt a particle moves a distance dx , as measured

in frame S . We obtain the corresponding distance dx' and time dt' in S' by taking differentials of Eqs. (37.21):

$$\begin{aligned}dx' &= \gamma(dx - u dt) \\dt' &= \gamma(dt - u dx/c^2)\end{aligned}$$

We divide the first equation by the second and then divide the numerator and denominator of the result by dt to obtain

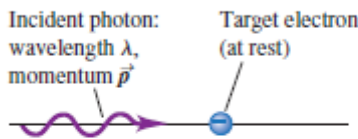
$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2} \frac{dx}{dt}}$$

Now dx/dt is the velocity v_x in S , and dx'/dt' is the velocity v'_x in S' , so we finally obtain the relativistic generalization

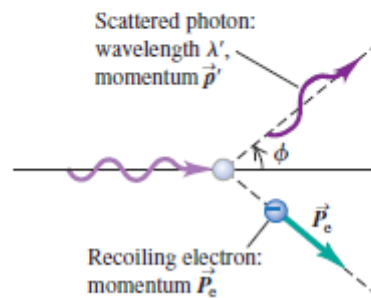
$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} \quad (\text{Lorentz velocity transformation}) \quad (37.22)$$

38.10 The photon model of light scattering by an electron.

(a) Before collision: The target electron is at rest.



(b) After collision: The angle between the directions of the scattered photon and the incident photon is ϕ .

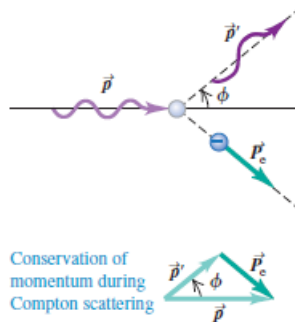


$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \quad (\text{Compton scattering}) \quad (38.7)$$

where m is the electron rest mass. In other words, λ' is greater than λ . The quantity h/mc that appears in Eq. (38.7) has units of length. Its numerical value is

$$\frac{h}{mc} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})} = 2.426 \times 10^{-12} \text{ m}$$

38.12 Vector diagram showing conservation of momentum in Compton scattering.



Compton showed that Einstein's photon theory, combined with the principles of conservation of energy and conservation of momentum, provides a beautifully clear explanation of his experimental results. We outline the derivation below. The electron recoil energy may be in the relativistic range, so we have to use the relativistic energy-momentum relationships, Eqs. (37.39) and (37.40). The incident photon has momentum \vec{p} , with magnitude p and energy pc . The scattered photon has momentum \vec{p}' , with magnitude p' and energy $p'c$. The electron is initially at rest, so its initial momentum is zero and its initial energy is its rest energy mc^2 . The final electron momentum \vec{p}_e has magnitude P_e , and the final electron energy is given by $E_e^2 = (mc^2)^2 + (P_e c)^2$. Then energy conservation gives us the relationship

$$pc + mc^2 = p'c + E_e$$

Rearranging, we find

$$(pc - p'c + mc^2)^2 = E_e^2 = (mc^2)^2 + (P_e c)^2 \quad (38.8)$$

We can eliminate the electron momentum \vec{P}_e from Eq. (38.8) by using momentum conservation. From Fig. 38.12 we see that $\vec{p} = \vec{p}' + \vec{p}_e$, or

$$\vec{P}_e = \vec{p} - \vec{p}' \quad (38.9)$$

By taking the scalar product of each side of Eq. (38.9) with itself, we find

$$P_e^2 = p^2 + p'^2 - 2pp' \cos \phi \quad (38.10)$$

We now substitute this expression for P_e^2 into Eq. (38.8) and multiply out the left side. We divide out a common factor c^2 ; several terms cancel, and when the resulting equation is divided through by (pp') , the result is

$$\frac{mc}{p'} - \frac{mc}{p} = 1 - \cos \phi \quad (38.11)$$

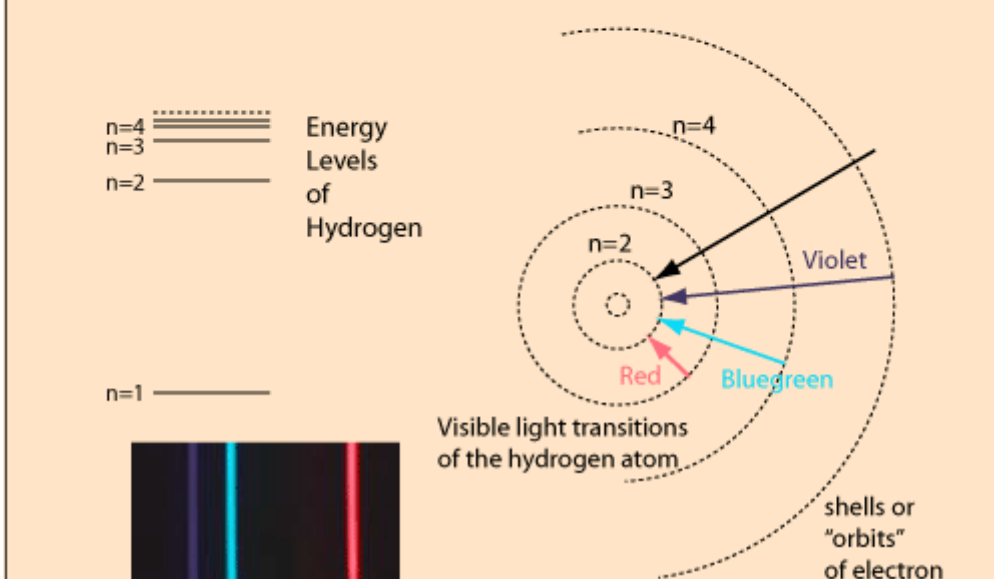
Finally, we substitute $p' = h/\lambda'$ and $p = h/\lambda$, then multiply by h/mc to obtain Eq. (38.7).

When the wavelengths of x rays scattered at a certain angle are measured, the curve of intensity per unit wavelength as a function of wavelength has two peaks (Fig. 38.13). The longer-wavelength peak represents Compton scattering. The shorter-wavelength peak, labeled λ_0 , is at the wavelength of the incident x rays and corresponds to x-ray scattering from tightly bound electrons. In such scattering processes the entire atom must recoil, so the m in Eq. (38.7) is the mass of the entire atom rather than of a single electron. The resulting wavelength shifts are negligible.

6)

Quantized Energy States

The electrons in free atoms can will be found in only certain discrete energy states. These sharp energy states are associated with the orbits or shells of electrons in an atom, e.g., a hydrogen atom. One of the implications of these quantized energy states is that only certain [photon energies](#) are allowed when electrons jump down from higher levels to lower levels, producing the [hydrogen spectrum](#). The [Bohr model](#) successfully predicted the energies for the hydrogen atom, but had [significant failures](#) that were corrected by solving the [Schrodinger equation](#) for the hydrogen atom.



Angular Momentum Quantization

In the [Bohr model](#), the wavelength associated with the electron is given by the [DeBroglie relationship](#)

$$\lambda = \frac{h}{mv}$$

and the [standing wave](#) condition that circumference = whole number of wavelengths. In the hydrogenic case, the number n is the principal quantum number.

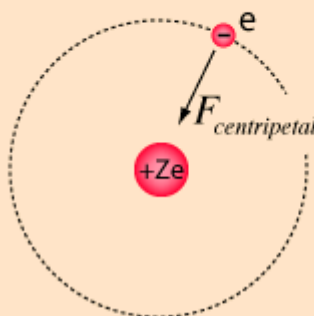
$$2\pi r = n\lambda_n$$

These can be combined to get an expression for the [angular momentum](#) of the electron in orbit. (Note that this assumes a circular orbit, a generally unwarranted assumption.)

$$L = mvr = \frac{hr}{\lambda} = \frac{hr}{\left[\frac{2\pi r}{n}\right]} = \frac{nh}{2\pi} \quad \text{Use in Bohr orbit}$$

Thus L is not only conserved, but constrained to discrete values by the quantum number n . This [quantization of angular momentum](#) is a crucial result and can be used in determining the Bohr orbit radii and Bohr energies.

Classical Electron Orbit



The orbit energy is negative because this is a bound state.

Newton's Second Law

$$= \frac{mv^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$

Potential Energy

$$U = \frac{-Ze^2}{4\pi\epsilon_0 r}$$

Kinetic Energy T

$$\text{so } T = \frac{mv^2}{2} = \frac{Ze^2}{8\pi\epsilon_0 r}$$

Total Energy

$$T + U = \frac{-Ze^2}{8\pi\epsilon_0 r}$$

This is the energy of a single electron in orbit around a bare nucleus.

In the [Bohr theory](#), this classical result was combined with the [quantization of angular momentum](#) to get an expression for [quantized energy levels](#).

Bohr Orbit

Combining the energy of the [classical electron orbit](#) with the [quantization of angular momentum](#), the Bohr approach yields expressions for the electron orbit radii and energies:

$\frac{mv^2}{2}$	$= \frac{(mvr)^2}{2mr^2}$	$= \frac{n^2 h^2}{8\pi^2 mr^2}$	$= \frac{Ze^2}{8\pi\epsilon_0 r}$	This is for hydrogenic atoms; the use of the atomic number Z is appropriate only if there is only one electron.
Kinetic energy of electron	expressed in terms of angular momentum.	Use quantization of angular momentum.	Set equal to total energy of classical orbit.	

Substitution for r gives the Bohr energies and radii:

$$E = -\frac{Z^2 m e^4}{8 n^2 h^2 \epsilon_0^2} = \frac{-13.6 Z^2}{n^2} \text{ eV} \quad r = \frac{n^2 h^2 \epsilon_0}{Z \pi m e^2} = \frac{n^2 a_0}{Z}$$

$a_0 = 0.0529 \text{ nm} = \text{Bohr radius}$

Failures of the Bohr Model

While the Bohr model was a major step toward understanding the quantum theory of the atom, it is not in fact a correct description of the nature of electron orbits. Some of the shortcomings of the model are:

1. It fails to provide any understanding of why certain spectral lines are brighter than others. There is no mechanism for the calculation of [transition probabilities](#).
2. The Bohr model treats the electron as if it were a miniature planet, with definite radius and momentum. This is in direct violation of the [uncertainty principle](#) which dictates that position and momentum cannot be simultaneously determined.

The Bohr model gives us a basic conceptual model of electrons orbits and energies. The precise details of spectra and charge distribution must be left to quantum mechanical calculations, as with the Schrodinger equation.

36.4 A PARTICLE IN A BOX

Consider a particle of mass m confined in a square potential well, that is, in a box, of width L . For $0 \leq x \leq L$, the potential energy $U(x) = 0$. Outside the well the potential energy is infinite, so there is no chance of finding the particle there. Thus the wave function must satisfy the boundary conditions $\psi(0) = \psi(L) = 0$. The Schrödinger wave equation is thus

$$\frac{d^2\psi}{dx^2} + \frac{\hbar^2}{2m}(E - 0)\psi = 0 \quad \text{or} \quad \frac{d^2\psi}{dx^2} + k^2\psi = 0$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

The solution is $\psi(x) = A \sin kx + B \cos kx$. Since $\psi(0) = 0 = 0 + B$, $B = 0$. Thus $\psi(x) = A \sin kx$. Since $\psi(L) = A \sin kL = 0$, $kL = n\pi$, where $n = 1, 2, 3, \dots$. The solution for the wave function is

$$\boxed{\psi(x) = A \sin\left(\frac{n\pi x}{L}\right) \quad n = 1, 2, 3, \dots} \quad (36.6)$$

The wavelength of these standing waves is $\lambda = 2L/n$, and the momentum of the particle is $p = \hbar/\lambda = n(\hbar/2L)$. This means that the energy is quantized because

$$E = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Thus

$$\boxed{E_n = \frac{\hbar^2}{8mL^2}n^2 \quad n = 1, 2, 3, \dots} \quad (36.7)$$

The wave function of Eq. 36.6 is normalized by the condition $\int_0^L \psi^2 dx = 1$:

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1, \quad A^2 \int_0^L \frac{1}{2} \left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx = \frac{1}{2}LA^2 = 1, \quad \text{so } A = \sqrt{\frac{2}{L}}$$

$$\boxed{\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad n = 1, 2, 3, \dots \quad \text{normalized wave function}} \quad (36.8)$$

41.4 The Zeeman Effect

The **Zeeman effect** is the splitting of atomic energy levels and the associated spectral lines when the atoms are placed in a magnetic field (Fig. 41.11). This effect confirms experimentally the quantization of angular momentum. The discussion in this section, which assumes that the only angular momentum is the *orbital* angular momentum of a single electron, also shows why we call m_l the magnetic quantum number.

Atoms contain charges in motion, so it should not be surprising that magnetic forces cause changes in that motion and in the energy levels. As early as the middle of the 19th century, physicists speculated that the sources of visible light might be vibrating electric charge on an atomic scale. In 1896 the Dutch physicist Pieter Zeeman was the first to show that in the presence of a magnetic field, some spectral lines were split into groups of closely spaced lines (Fig. 41.12). This effect now bears his name.

Magnetic Moment of an Orbiting Electron

Let's begin our analysis of the Zeeman effect by reviewing the concept of *magnetic dipole moment* or *magnetic moment*, introduced in Section 27.7. A plane current loop with vector area \vec{A} carrying current I has a magnetic moment $\vec{\mu}$ given by

$$\vec{\mu} = I\vec{A} \quad (41.27)$$

When a magnetic dipole of moment $\vec{\mu}$ is placed in a magnetic field \vec{B} , the field exerts a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ on the dipole. The potential energy U associated with this interaction is given by Eq. (27.27):

$$U = -\vec{\mu} \cdot \vec{B} \quad (41.28)$$

Now let's use Eqs. (41.27) and (41.28) and the Bohr model to look at the interaction of a hydrogen atom with a magnetic field. The orbiting electron with speed v is equivalent to a current loop with radius r and area πr^2 . The average current I is the average charge per unit time that passes a given point of the orbit. This is equal to the charge magnitude e divided by the time T for one revolution, given by $T = 2\pi r/v$. Thus $I = ev/2\pi r$, and from Eq. (41.27) the magnitude μ of the magnetic moment is

The ratio of the magnitude of $\vec{\mu}$ to the magnitude of \vec{L} is $\mu/L = e/2m$ and is called the *gyromagnetic ratio*.

In the Bohr model, $L = nh/2\pi = n\hbar$, where $n = 1, 2, \dots$. For an $n = 1$ state (a ground state), Eq. (41.30) becomes $\mu = (e/2m)\hbar$. This quantity is a natural unit for magnetic moment; it is called one **Bohr magneton**, denoted by μ_B :

$$\mu_B = \frac{e\hbar}{2m} \quad (\text{definition of the Bohr magneton}) \quad (41.31)$$

Evaluating Eq. (41.31) gives

$$\mu_B = 5.788 \times 10^{-5} \text{ eV/T} = 9.274 \times 10^{-24} \text{ J/T or A} \cdot \text{m}^2$$

magnitude μ_B . But the Schrödinger picture tells us that such a ground-state electron is in an s state with zero angular momentum, so the orbital magnetic moment must be *zero*! To get the correct results, we must describe the states by using Schrödinger wave functions.

It turns out that in the Schrödinger formulation, electrons have the same ratio of μ to L (gyromagnetic ratio) as in the Bohr model—namely, $e/2m$. Suppose the magnetic field \vec{B} is directed along the $+z$ -axis. From Eq. (41.28) the interaction energy U of the atom's magnetic moment with the field is

$$U = -\mu_z B \quad (41.32)$$

where μ_z is the z -component of the vector $\vec{\mu}$.

Now we use Eq. (41.30) to find μ_z , recalling that e is the *magnitude* of the electron charge and that the actual charge is $-e$. Because the electron charge is negative, the orbital angular momentum and magnetic moment vectors are opposite. We find

$$\mu_z = -\frac{e}{2m}L_z \quad (41.33)$$

For the Schrödinger wave functions, $L_z = m_l\hbar$, with $m_l = 0, \pm 1, \pm 2, \dots, \pm l$, so Eq. (41.33) becomes

$$\mu_z = -\frac{e}{2m}L_z = -m_l \frac{e\hbar}{2m} \quad (41.34)$$

CAUTION Two uses of the symbol m Be careful not to confuse the electron mass m with the magnetic quantum number m_l .

Finally, we can express the interaction energy, Eq. (41.32), as

$$U = -\mu_z B = m_l \frac{e\hbar}{2m} B \quad (m_l = 0, \pm 1, \pm 2, \dots, \pm l)$$

(orbital magnetic interaction energy) (41.35)

In terms of the Bohr magneton $\mu_B = e\hbar/2m$, we can write Eq. (41.35) as

$$U = m_l \mu_B B \quad (\text{orbital magnetic interaction energy}) \quad (41.36)$$

This is equal to the charge magnitude e divided by the time T for one revolution, given by $T = 2\pi r/v$. Thus $I = ev/2\pi r$, and from Eq. (41.27) the magnitude μ of the magnetic moment is

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2} \quad (41.29)$$

We can also express this in terms of the magnitude L of the orbital angular momentum. From Eq. (10.28) the angular momentum of a particle in a circular orbit is $L = mvr$, so Eq. (41.29) becomes

$$\mu = \frac{e}{2m} L \quad (41.30)$$

10)

42.5 Free-Electron Model of Metals

Studying the energy states of electrons in metals can give us a lot of insight into their electrical and magnetic properties, the electron contributions to heat capacities, and other behavior. As we discussed in Section 42.3, one of the distinguishing features of a metal is that one or more valence electrons are detached from their home atom and can move freely within the metal, with wave functions that extend over many atoms.

The **free-electron model** assumes that these electrons are completely free inside the material, that they don't interact at all with the ions or with each other, but that there are infinite potential-energy barriers at the surfaces. The idea is that a typical electron moves so rapidly within the metal that it "sees" the effect of the ions and other electrons as a uniform potential-energy function, whose value we can choose to be zero.

We can represent the surfaces of the metal by the same cubical box that we analyzed in Section 41.2 (the three-dimensional version of the particle in a box studied in Section 40.2). If the box has sides of length L (Fig. 42.20), the energies of the stationary states (quantum states of definite energy) are

$$E_{n_X, n_Y, n_Z} = \frac{(n_X^2 + n_Y^2 + n_Z^2)\pi^2\hbar^2}{2mL^2} \quad (n_X = 1, 2, 3, \dots; n_Y = 1, 2, 3, \dots; n_Z = 1, 2, 3, \dots) \quad (42.10)$$

Each state is labeled by the three positive-integer quantum numbers (n_X, n_Y, n_Z) .

Average Free-Electron Energy

We can calculate the *average* free-electron energy in a metal at absolute zero by using the same ideas that we used to find E_{F0} . From Eq. (42.17) the number dN of electrons with energies in the range dE is $g(E)f(E) dE$. The energy of these electrons is $E dN = Eg(E)f(E) dE$. At absolute zero we substitute $f(E) = 1$ from $E = 0$ to $E = E_{\text{F0}}$ and $f(E) = 0$ for all other energies. Therefore the total energy E_{tot} of all the N electrons is

$$E_{\text{tot}} = \int_0^{E_{\text{F0}}} Eg(E)(1) dE + \int_{E_{\text{F0}}}^{\infty} Eg(E)(0) dE = \int_0^{E_{\text{F0}}} Eg(E) dE$$

The simplest way to evaluate this expression is to compare Eqs. (42.15) and (42.19), noting that

$$g(E) = \frac{3NE^{1/2}}{2E_{\text{F0}}^{3/2}}$$

Substituting this expression into the integral and using $E_{\text{av}} = E_{\text{tot}}/N$, we get

$$E_{\text{av}} = \frac{3}{2E_{\text{F0}}^{3/2}} \int_0^{E_{\text{F0}}} E^{3/2} dE = \frac{3}{5}E_{\text{F0}} \quad (42.21)$$

At absolute zero the average free-electron energy equals $\frac{3}{5}$ of the Fermi energy.