$E = R(1-T)$

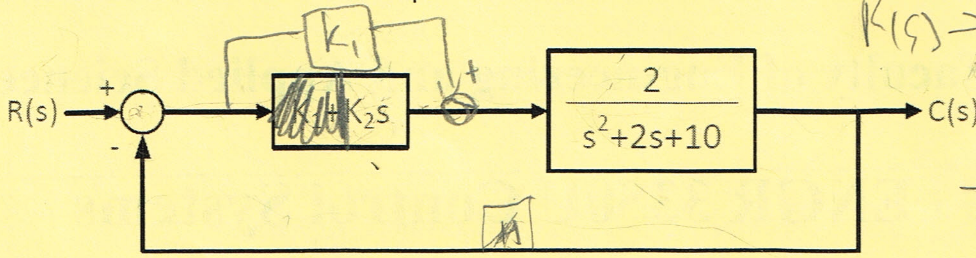
$\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

Problem 1 (30 marks)

28/30

The closed–loop control system, shown in the figure below, is used to control the movement of an industrial robot. In order to improve its performance, a controller action, with the parameters K_1 and K_2, has been added in the forward path.

$R(s) \rightarrow \boxed{T}$



$T(s) = \dfrac{C(s)}{R(s)}$

1. Obtain the closed-loop transfer function of the robot, C(s) / R(s).
2. Find the numerical values of the parameters if the system response to a unit step input has the *settling time* of **2** seconds and the *peak time* of **0.9063** seconds.
3. Determine the numerical value(s) of the pole(s) and zero(s) of the system.

1. $R(s) \xrightarrow{+} \bigcirc \rightarrow \boxed{\dfrac{(k_1+k_2 s)2}{s^2+2s+10}} \rightarrow C(s)$ (with $-$ feedback) $\Rightarrow R(s) \rightarrow \boxed{\dfrac{(k_1+k_2s)2/(s^2+2s+10)}{1+(k_1+k_2s)2/(s^2+2s+10)}} \rightarrow C(s)$

$$T = \frac{(k_1+k_2 s)2}{s^2+2s+10+(k_1+k_2 s)2}$$

$$\boxed{\therefore\ T(s) = \frac{(k_1+k_2 s)2}{s^2+(2+2k_2)s+10+2k_1}}$$ 1

2. T is similar to 2nd order function $\dfrac{\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$t_s \approx \dfrac{4}{\xi\omega_n}$

$\xi\omega_n = 2 \Rightarrow \omega_n = \dfrac{2}{\xi}$

&

$\omega_n\sqrt{1-\xi^2} = \dfrac{\pi}{0.9063} = 3.4664$

$\dfrac{3.4664}{2}\xi = \sqrt{1-\xi^2}$

$(1.7332\xi)^2 = |1-\xi^2|$

$3.0040\xi^2 = 1-\xi^2$

$\sqrt{\xi^2} = \sqrt{\dfrac{1}{4.0040}}$

$\boxed{\xi = 0.49975}$

$\omega_n \therefore 10+2k_1 = (4.0020)^2$ & $2\xi\omega_n = 2+2k_2 \Rightarrow \xi\omega_n = 1+k_2$

$\boxed{\therefore k_1 = 3.0080}$ 2

$k_2 = \xi\omega_n - 1 = 2$

$\boxed{\therefore k_2 = 1}$ 2

3. Characteristic equation (assuming $k_1 \approx 3$) is: $s^2+8s+12=0$

$\Rightarrow (s+2)(s+6)=0 \therefore$ poles exist @ $s=-2$ & s 3

$(k_1+k_2 s)2 = 0$

$6+2s=0$ 3

$s=-3$ $\therefore$ Zero exists @ $s=-$

Problem 2 (30 marks)

30/30

a) The transfer function of a closed–loop control system is given as:

$$\frac{Y(s)}{R(s)} = \frac{Ks + 40K}{s^3 + 11s^2 + (K+10)s + 40K}$$

1. Find the range of variations of the gain K such that the system remains stable.
2. What is the value of K for marginal stability?

b) Determine the number of positive root(s) of the polynomial given below:

$$s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9 = 0$$

a)

s^3	1	$k+10$	0
s^2	11	$40k$	0
s^1	$\frac{11k+110-40k}{11}$	0	
s^0	$40k$		

$k + 10 - \frac{40k}{11} > 0$

$\left(1 - \frac{40}{11}\right)k > -10$

$-\frac{29}{11}k > -10$

$\therefore k < \frac{10 \cdot 11}{29} = \frac{110}{29}$

$K+10 > 0$

& $40k > 0$

$\therefore k > 0$

lowe

1. $\boxed{0 < k < \frac{110}{29}}$

$\therefore$ system is stable in range of

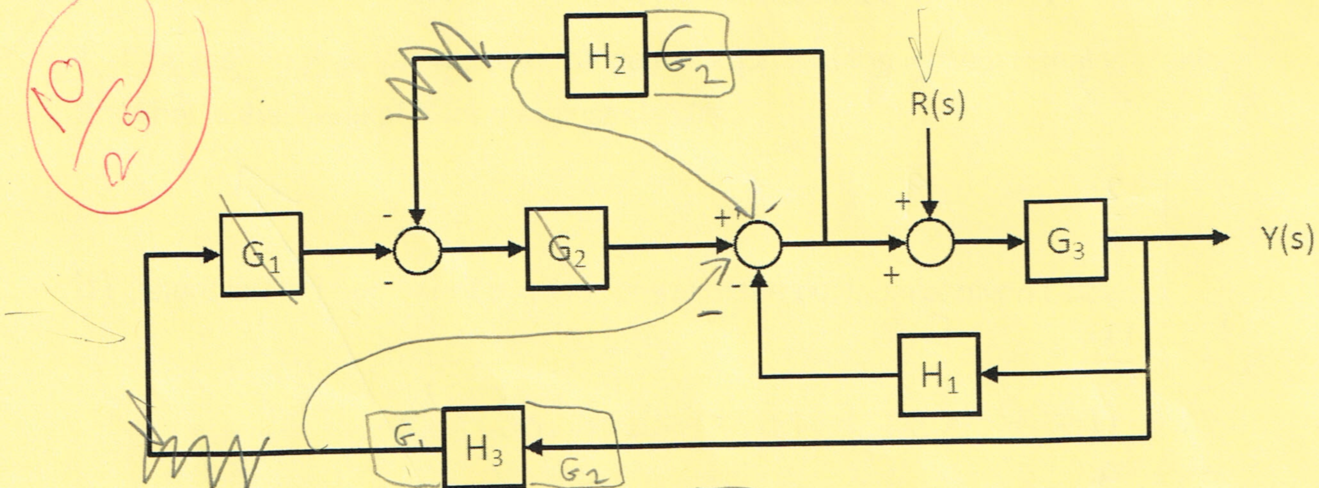
2. if $k = \frac{110}{29} \approx 3.79$, the system is marginally stab

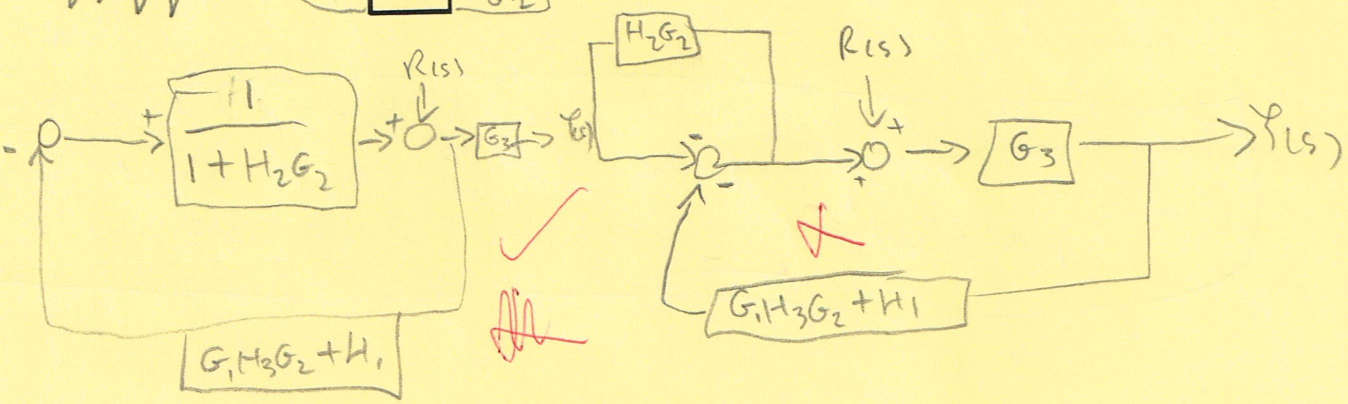
see next page for b)

Problem 3 (25 Marks)

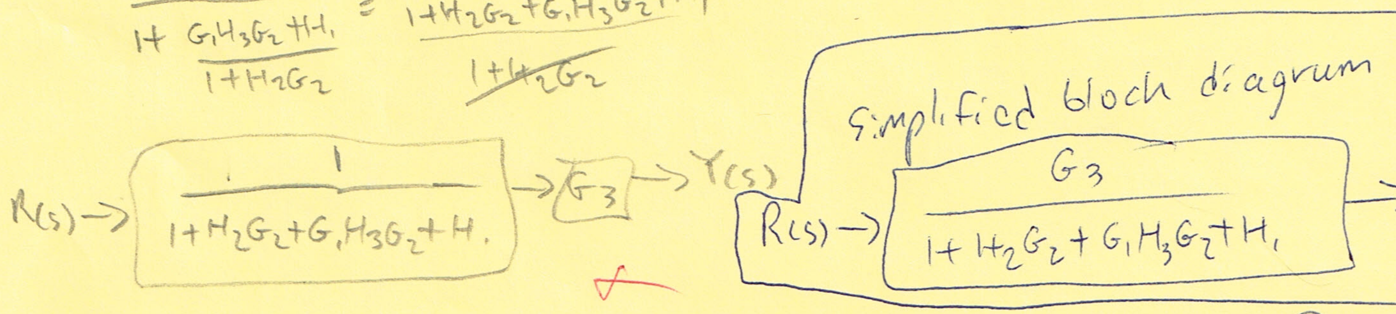
10/25

1. Use the "block diagram reduction" method to simplify the block diagram shown below.
2. Determine the closed–loop transfer function $T(s) = Y(s) / R(s)$ **in its most simple form** and in terms of G_1, G_2, G_3, H_1, H_2 and H_3.





$$\frac{1/1+H_2G_2}{\frac{1+G_1H_3G_2+H_1}{1+H_2G_2}} = \frac{\cancel{1/1+H_2G_2}}{\frac{1+H_2G_2+G_1H_3G_2+H_1}{\cancel{1+H_2G_2}}}$$



$$\therefore T(s) = \frac{G_3}{1+G_2H_2+G_1G_2H_3+H_1}$$

① ②

Problem 4 (15 marks)

2/15

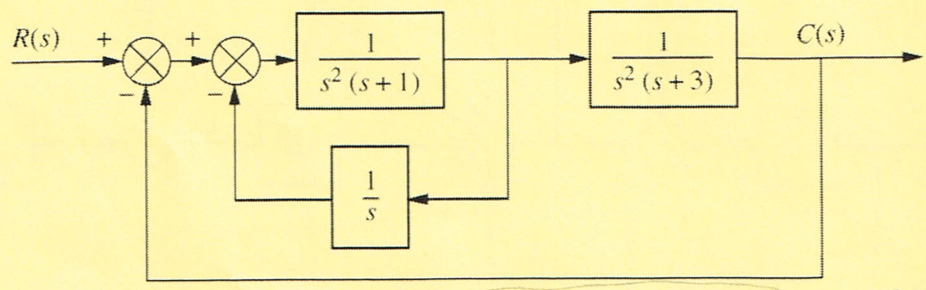
The block diagram of a closed–loop control system is shown in the figure below.

a) Determine the closed-loop transfer function, $T(s) = C(s) / R(s)$.

b) Specify both the 'type' and 'order' of the system.

c) Find the steady-state error of the control system for a ramp input function of $10t$.

$$\frac{\frac{1}{s^2(s+1)}}{1+\frac{s}{s^2(s+1)}}$$



1	5
2	4
3	2.5
2.33	1
17/4	

$$\frac{\cancel{\frac{1}{s^2(s+1)}}}{s^2(s+1)+s}$$

$$\frac{1}{s^2(s+1)+s}$$

$$R(s) \to \boxed{\frac{1}{(s^2(s+1)+s)(s^2+s+3)}} \to C(s)$$

$(s^3+s^2+s)(s^2$

$s^5+s^4+3s^3+s^3$

$+s^3+s^2+3s$

$= s^5+2s^4+5s^3+$

a) $\boxed{T(s) = \frac{1}{s^5+2s^4+5s^3+4s^2+3s+1}}$

$$\frac{1/x}{1+1/x}\cdot\frac{x}{x} = \frac{1}{x+1}$$

$T(s) = _{s\to0} = \frac{1}{0+1}$

b) $\boxed{\text{Order } 5}$ $\boxed{\text{type: stable}}$

c) $R(s) = \frac{10}{s^2}$ from table $E(s) = \frac{10}{s^2}(1-T(s))$

$\boxed{T(0) =}$

to find steady state error → $\lim_{s\to0} s\cdot E(s) = \lim_{s\to0}\frac{10}{s}(1-1)$

$= \lim_{s\to0}\frac{0}{s} \Rightarrow \infty$

$\boxed{\therefore \text{ steady-state error } = \infty}$

Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$

Maximum percent overshoot, M_p: $$P.O. = 100 e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

W_n

$\zeta = \frac{C}{C_c}$

Peak time: $$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

Settling time: $$T_s \cong \frac{4}{\zeta\omega_n}$$