Opgaven 12: oefening 8

$$g(n) = \alpha^2 \mathbb{H}(n) \leftrightarrow G^{DTFT}(\omega - \theta) = \frac{1}{1 - \alpha e^{-i\omega}}$$

En als we ons beperken tot $\omega \in [0, 2\pi]$

$$q(n) = \sum_{k=0}^{N-1} G^{DFR}(k) e^{ik\frac{2\pi}{N}n} \leftrightarrow Q^{DTFT}(\omega) = \sum_{k=0}^{N-1} G^{DFR}(k) 2\pi\delta(\omega - k\frac{2\pi}{N})$$

$$\begin{aligned}
X^{DTFT}(\omega) &= \sum_{n=-\infty}^{\infty} q(n)g(n)e^{-i\omega n} \\
&= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi}\int_0^{2\pi} Q^{FT}(\theta)e^{i\theta n} d\theta\right] g(n)e^{-i\omega n} \\
&= \frac{1}{2\pi}\int_0^{2\pi} Q^{DTFT}(\theta)\left[\sum_{n=-\infty}^{\infty} g(n)e^{-i(\omega-\theta)}\right] d\theta \\
&= \frac{1}{2\pi}\int_0^{2\pi} Q^{DTFT}(\theta) G^{DTFT}(\omega - \theta) d\theta \\
&= \frac{1}{2\pi}\int_0^{2\pi} \sum_{k=0}^{N-1} G^{DFR}(k) 2\pi\delta(\theta - k\frac{2\pi}{N}) \frac{1}{1 - \alpha e^{-i(\omega-\theta)}} d\theta \\
&= \sum_{k=0}^{N-1} G^{DFR}(k) \int_0^{2\pi} \delta(\theta - k\frac{2\pi}{N}) \frac{1}{1 - \alpha e^{-i(\omega-\theta)}} d\theta \\
&= \sum_{k=0}^{N-1} G^{DFR}(k) \frac{1}{1 - \alpha e^{-i(\omega - k\frac{2\pi}{N})}}
\end{aligned}$$

Waaruit volgt dat:

$$\begin{aligned}
N &= 4 \\
\alpha &= \frac{1}{4} \\
G^{DFR}(k) &= \left(\frac{1}{2}\right)^k
\end{aligned}$$

En aangezien $G(k) \neq \bar{G}(-k)$ is $x(n)$ niet reel.