

hyperbolic function definition

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\coth(x) = \frac{1}{\tanh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\operatorname{cosech}(x) = \frac{1}{\sinh(x)}$$

Limmit rules

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

M and k is a constant then

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = L \pm M$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = LM$$

$$\lim_{x \rightarrow a} [kf(x)] = kL$$

$$\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{L}{M}, M \neq 0$$

Trig identities

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x)$$

$$\frac{d}{dx} \coth(x) = -\operatorname{cosech}^2(x)$$

$$\frac{d}{dx} \operatorname{sech}(x) = -\operatorname{sech}(x)\tanh(x)$$

$$\frac{d}{dx} \operatorname{cosech}(x) = -\operatorname{cosech}(x)\coth(x)$$

$$\cosh(-x) = \cosh(x)$$

$$\sinh(-x) = -\sinh(x)$$

$$\sinh(2x) = 2 \sinh(x) \cosh(x)$$

$$\cosh(2x) = \cosh^2(x) + \sinh^2(x) = 1 + 2 \sinh^2(x) = 2 \cosh^2(x) - 1$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$(\ln(y) = x) = (y = e^x)$$

$$e^{(\log_e x)} = x$$

$$\log_a(a^x) = x$$

$$(a+b)(a-b) = (a^2 - b^2)$$

A function continuous if

- $f(x)$ exists
- $\lim_{x \rightarrow c} f(x)$ exists
- These two quantities are equal

Conditions of continuous functions.

Continuity

If $f(x)$ and $g(x)$ are continuous then the following are continuous

- $f(x) + g(x)$
- $cf(x)$
- $f(x)g(x)$ and
- $f(x)/g(x)$ when $g(x)$ is not equal to 0

The following types of functions are

continuous for all points in there domain

- Polynomials
- Trigonometric Functions
- Exponential functions
- Logarithmic functions
- Root functions

$$\tan^{-1}(x) = z$$

$$x = \tan(z)$$

$$\frac{d}{dx}(x) = \frac{d}{dz} \tan(z)$$

$$1 = \sec^2(z) \frac{dz}{dx}$$

$$\frac{1}{\sec^2(z)} = \frac{dz}{dx}$$

$$\frac{1}{1 + \tan^2(z)} = \frac{dz}{dx}$$

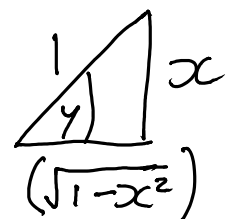
$$\frac{1}{1 + x^2} = \frac{dz}{dx}$$

$$y = \sin^{-1}(x)$$

$$x = \sin(y)$$

$$\frac{d}{dx}(x) = \frac{d}{dy}(\sin(y))$$

$$1 = \cos(y) \frac{dy}{dx}$$



Limits tricks

- Squeeze theorem
 - Find a function either side
- L'Hopitals rule (only works when limit is in indeterminate form).
 - $0/0, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \frac{\infty}{-\infty}, \frac{-\infty}{-\infty}$
- Factorising and cancelling common factors
- Multiply by conjugate and cancel common factors

$\frac{d}{dx}(cf(x)) = cf'(x)$ c is any constant	$(f(x) \pm g(x))' = f'(x) \pm g'(x)$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(c) = 0$ c is any constant
$\frac{d}{dx}(fg) = \frac{d}{dx}(f) \times g + f \times \frac{d}{dx}(g)$	$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{d}{dx}(f) \times g - f \times \frac{d}{dx}(g)}{g^2}$
$\frac{d}{dx}(f(g(x))) = f'(g(x)) \times g'(x)$	
$\frac{d}{dx}(e^{g(x)}) = g'(x)e^{g(x)}$	$\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\cos(x) = -\sin(x)$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	
$\frac{d}{dx}(a^x) = a^x \ln(a)$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x \neq 0$	$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$
$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a}, x > 0$	

A function is continuous if it is derivable but does **not** have to be derivable if it is continuous

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Quadratic equation

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

