

Day 20

Last week we learned to create scatterplots to assess potential associations between two quantitative variables. We described the association by telling about Form, direction, strength, unusual features. Then we learned how to calculate correlation.

Describe the strength of the association between two variables as long as these conditions are met:
quantitative, linear, no outliers

Remember, correlation measures the strength of association between x and y by measuring how closely the points of a scatterplot hug a line that goes through all the points that are an equal number of standard deviations away from the mean for both variables. Today we seek to figure out:

How can we use the correlation coefficient to estimate the y value for a given x value?

We call the estimate made from a model the predicted value, and write it as \hat{y} (y -hat) to distinguish it from the true value of y .

If there is a perfect correlation (that is, $r = -1$ or $r = 1$) then we can perfectly predict y from x .

- $r = 1$ If x goes up 1 SD from average, then y goes up 1 SD from average. All points lie on the SD line.
- $r = .8$ If x increases 1 SD, then y increases only 0.8 SD on the average.
- $r = .5$ If x increases 1 SD, then y increases only 0.5 SD on the average.
- $r = -.3$ If x increases 1 SD, then y decreases only 0.3 SD on the average.
- $r = 0$ If x increases 1 SD, then y decreases only 0 SD on the average.

Regression estimates the average value for the response variable (y) corresponding to each value of the explanatory variable (x).

1 SD increase in x means only an r SD increase in y .

Example 1: A large class took two exams with the following results:

$$\bar{x}_{\text{exam1}} = 80 \quad s_{\text{exam1}} = 10, \quad \bar{x}_{\text{exam2}} = 70 \quad s_{\text{exam2}} = 15, \quad r = .6$$

- a) Suppose a student is picked at random. What would be your best guess for what he got on exam 2?

Don't know anything \rightarrow guess any number

- b) Now suppose you're told that the student got a 90 on Exam 1, what would be your best guess for his score on Exam 2?

$$\text{Z-score} = \frac{90 - 80}{10}$$

$$\text{Z-score} = 1$$

$$r = 0.6 = 0.6$$

$$\begin{aligned} \text{Exam 2} \\ 0.6 &= \frac{\hat{y} - 70}{15} \\ \hat{y} &= 79 \end{aligned}$$

- c) A student got a 65 on Exam 1, but hasn't taken Exam 2 yet. What would you predict his score to be?

$$\text{Exam 1} \\ \text{Z-score} = \frac{65 - 80}{10}$$

$$\text{Z-score} = -1.5$$

$$r = -1.5 \cdot 0.6 = -0.9$$

$$\begin{aligned} \text{Exam 2} \\ -0.9 &= \frac{\hat{y} - 70}{15} \\ \hat{y} &= 56.5 \end{aligned}$$

Note: The estimated z-score is always closer to 0 than the given z-score unless perfect correlation.

Regression to the mean

Because the correlation is always less than 1.0 in magnitude, each predicted \hat{y} tends to be fewer SD from its mean than its corresponding x was from its mean.

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Complete the rest of the problems on the handouts until you see Day 20 Stop