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Example 2: Scores on the Math SAT I and Math SAT II can both be approximated with a normal model. If a student was in the 80th percentile on the Math SAT I, predict what his percentile on the SAT II will be under the following conditions.

a) $r = 1$

SAT I → SAT II

80th% 80th%

b) $r = 0$

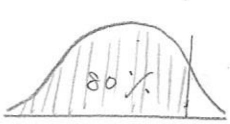
SAT I → SAT II

80th% 50th%

because $r=0$; know nothing; guess mean

c) $r = .8$

SAT I



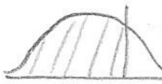
80%

invNorm (0.8) = 0.8416

SAT I: z-score = 0.8416

r: 0.8416 · 0.8 = .6733

SAT II: N(0,1) normal cdf(0.6733) = 75%



0.6733

Example 3: Scores on the Math SAT average 500 with a SD of 100. An SAT prep school enrolls about 100 people who scored 300 their first time tested. On their second test, their average was 350. The prep course administrators claim this 50-point gain was due to their course. Overall, 5000 people took the test twice and the correlation between scores on the first test and scores on the second (for all 5000 people) was 0.7. Is the 50-point gain evidence that the prep course helps or is the improvement simply due to the regression effect?

1st time

$$z\text{-score} = \frac{300-500}{100}$$

z-score = −2

r: $-2 \cdot 0.7 = -1.4$

2nd time

$$-1.4 = \frac{\hat{y}-500}{100}$$

$\hat{y} = 360$; the 50-point gain is just due to the regression effect

Example 4: A study of the heights of mothers and their sons produced the following results:

$\bar{x}_{mother} = 65''$ $s_{mother} = 2''$, $\bar{x}_{son} = 69''$ $s_{son} = 3''$, $r = 0.5$

a) Estimate the average height of those sons whose mothers are 69" tall.

mother

$$z_y = \frac{69-65}{2}$$

$z_y = 2$

r: $2 \cdot 0.5 = 1$

son

$$1 = \frac{\hat{y}-69}{3}$$

$\hat{y} = 72$

72 in. tall

b) Estimate the average height of those mothers whose sons are 72" tall.

Son

$$z_y = \frac{72-69}{3}$$

$z_y = 1$

r: $1 \cdot 0.5 = 0.5$

Mother

$$0.5 = \frac{\hat{y}-65}{2}$$

$\hat{y} = 66$

66 in. tall