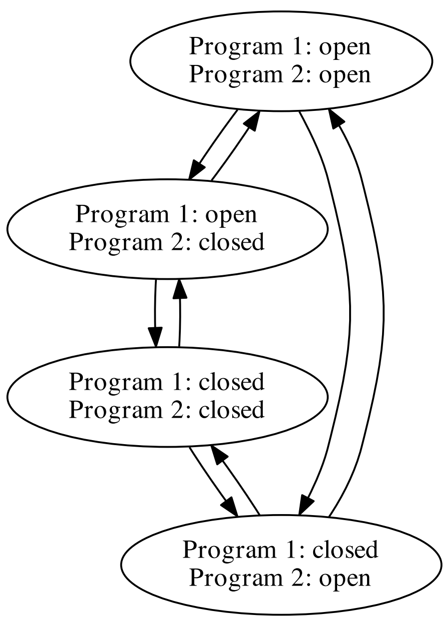
Submission

Make a single PDF file containing your answers to the following four questions. Upload it to the CO519 Moodle page by the end of Friday, 1 November, 2013. Failure to follow these instructions will incur a penalty of at least 10% of the marks.

Question 1

Consider the situation where two different programs (for example, emacs and vi or BlueJ and Eclipse) might try to use the same file at the same time. Each program can have the file open or closed. A program can only open the file when it does not already have it opened, and it can only close the file when it does have it opened. This question asks you to model this situation using propositional logic. The system looks like this, diagrammatically:



Unlike the traffic light example, each configuration has more than one possible next configuration.

1. (2 marks) We want to model how the status of the file (opened/closed) can change through time for both programs. Choose some propositional variable names for this purpose and clearly state what each represents. (Hint: Use one group of variables that do not end with $'$ to represent the

system *now* and another group that do end in $'$ to represent the system after one of the programs opens or closes the file.)

2. (2 marks) If both programs have the file opened at the same time, their updates to the file might conflict. Write a propositional logic formula using the variables from part 1 to express that both programs do not have the file open right now.

3. (2 marks) Again using the variables from part 1, write a propositional logic formula that precisely describes how the system can transition. Assume that the programs do not check whether they might interfere with each other. (Hint: Because the traffic light had only one possible next state, its specification did not use disjunction ($\vee$). Here, disjunction will help model the situation where there are two possible transitions.)

Question 2

There is a monkey in a room that contains a box on the floor and a banana hanging from the ceiling. The monkey cannot reach the banana without standing on the box, but fortunately for the monkey, it can move the box around the room and climb onto and off of the box. We can use propositional logic to model this situation: either the monkey is standing on the box or not, either the box is under the banana or not, and either the monkey has the banana or it is still on the ceiling. The monkey can take the banana when it is standing on the box under the banana.

1. (2 marks) Introduce propositional variables to model this situation. Clearly state what each one represents.

2. (2 marks) Using the variables from part 1, write a propositional logic formula that precisely describes how the room can move from configuration to configuration. Assume that the monkey can move the box only when not standing on it. (Hint: see the hint for Question 1 part 3 above.)

3. (2 marks) Again, using the variables from part 1, write a propositional logic formula that states that the monkey can take the banana if and only if the box is under the banana and the monkey is standing on the box. In other words, there can only be a transition from a banana-on-the-ceiling configuration to a monkey-has-the-banana configuration if the box is currently under the banana and the monkey is currently standing on it.

4. (4 marks) Using a decision tree, verify that the model of the system only lets the monkey take the banana when it is possible. In other words, verify that the formula from part 2 implies the formula from part 3.

Question 3

Chess in FOL. We will use the following predicates (also called relations)

- has-white-king(x, y): the white king is at space (x, y).
- has-white-piece(x, y): there is a white piece at space (x, y).
- has-black-piece(x, y): there is a black piece at space (x, y).
- kings-move(x_1, y_1, x_2, y_2): the space (x_1, y_1) is next to (x_2, y_2), which are all of the spaces that a king could potentially move to.
- knights-move(x_1, y_1, x_2, y_2): the space (x_2, y_2) is a knight's move from (x_1, y_1).
- has-knight(x, y): the space (x, y) has a knight of either colour.

The king can move to any space that is a king's move away, unless that space is already occupied by a piece of the same colour.

1. (1 mark) Write a formula in first-order logic, using only the relations above, that states that all of the black pieces on the board are knights.
2. (1 mark) Write a formula in first-order logic, using only the relations above, that states that all of the white pieces on the board are either knights or kings.
3. (2 marks) Write a formula in first-order logic, using only the relations above, that states that there is at most 1 white king on the board. (Hint: you will need to use the = operator.)
4. (2 marks) Assume that the only black pieces on the board are knights. Write a formula in first-order logic, using only the relations above, that is true when there is a white king in check.
5. (2 marks) A king is checkmated if it is both

    - in check, and
    - either it cannot move, or for all of the spaces that it can move to, it would be in check there too.

    Assume that the only black pieces on the board are knights. Suppose we have a relation would-be-check(x, y) that is true iff the white king would be in check on the given space. Write a formula in first-order logic, using only the relations above and would-be-check, that is true when the white king is checkmated.

Question 4

(4 marks) Use the DPLL algorithm to prove that this CNF formula is unsatisfiable.

$$(x \vee \neg y \vee \neg z) \wedge (\neg x \vee \neg y) \wedge (y \vee z) \wedge (\neg x \vee \neg z) \wedge (y \vee \neg z) \wedge (\neg y \vee z)$$

Hint: start with a case split on z.