7.1.3 Final Exam: Semester Exam (Teacher-Scored)

Test

Precalculus Sem 2 (S2062436)

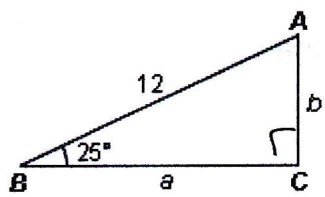
Andrew Bui

Points possible: 100

Date: ________

Answer the following questions using what you've learned from this unit. Write your responses in the space provided.

1. Consider the right triangle *ABC* given below:



Part I: Find the length of side *b* to two decimal places. **(2 points)**

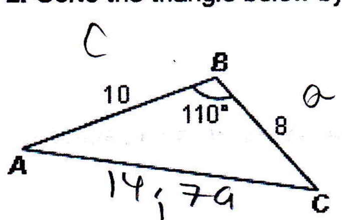
$$\frac{\sin C}{c} = \frac{\sin B}{b} \qquad \frac{\sin 90}{12} = \frac{\sin 25}{b}$$

Part II: Find the length of side *a* to two decimal places using the method of your choice. **(2 points)**

$$\frac{b\sin 90}{\sin 90} = \frac{12\sin 25}{\sin 90}$$

$$b = \frac{12\sin 25}{\sin 90} = \sim \boxed{5.07}$$

2. Solve the triangle below by finding all missing sides and/or angles.



side b

Part I: Find the length of side *AC*. Round your answer to the nearest hundredth. **(4 points)**

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 10^2 + 8^2 - 2(80)\cos 110$$

$$b^2 = 100 + 64 - 160\cos 110$$

$$b^2 = 216.72$$

$$\boxed{b = 14.79}$$

Part II: Find the measure of angle *C*. **(4 points)**

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{\sin 110}{14.79} = \frac{\sin C}{10}$$

$$10\sin 110 = 14.79\sin C$$

$$\frac{10\sin 110}{14.79} = \sin C$$

cont'd

$.635 = \sin C \quad \sin^{-1}(.635) = C \quad \boxed{39.45° = C}$

Part III: Find the measure of angle *A* using any method. **(2 points)**

$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \frac{\sin A}{8} = \frac{\sin 110}{14.79} \quad 8\sin 110 = 14.79 \sin A \quad \frac{8\sin 110}{14.79} = \sin A$

$.51 = \sin A$

$\sin^{-1}(.51) = A$

$\boxed{30.55 = A}$

3. Graph the function $y = -3\cos(2x) + 1$ on the axes below.

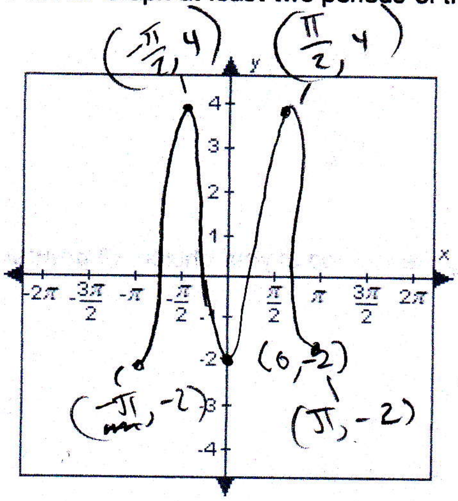
Part I: Identify the amplitude, vertical shift, and period of this function. **(6 points)**

amp: 3

v. shift: up 1

period: π

Part II: Graph at least two periods of this function. **(4 points)**



4. An object's motion is described by the equation $d = 3\sin(8\pi t) - 2$. The displacement, *d*, is measured in meters. The time, *t*, is measured in seconds. Answer the following questions.

Part I: What is the object's position (in meters) at $t = 0$? **(4 points)**

$d = (0) - 2$

$d = -2$

Part II: What is the object's maximum displacement (in meters) from its $t = 0$ position? **(2 points)**

3

Part III: How much time (in seconds) is required for one oscillation? **(2 points)**

$\frac{2\pi}{8\pi} = \frac{1}{4} s$

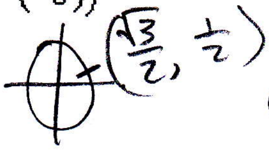
Part IV: What is the frequency (measured in Hz) of this oscillation? **(2 points)**

4

5. Evaluate each of the following. In each case, explain your thinking.

a. $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$ **(6 points)**

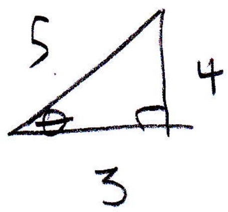
$\boxed{\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \approx .52}$



explanation:
$\cos\left(\frac{-\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \approx .52$

b. $\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

Part I: Draw a triangle with an angle whose tangent is $\frac{4}{3}$. **(2 points)**

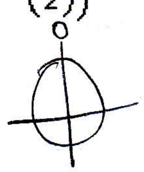


$\tan^{-1}(4/3) \approx 53.13$

Part II: Find the sine of angle θ. **(2 points)**

$\boxed{\sin(53.13) = .8}$

c. $\tan^{-1}\left(\cos\left(\frac{\pi}{2}\right)\right)$ **(6 points)**



$\boxed{\tan^{-1}(0) = 0, \pi}$

Exp: $\cos\left(\frac{\pi}{2}\right) = 0$. If $\tan\theta = 0$, then $\sin\theta = 0$. If $\sin\theta = 0$, $\theta = 0$ or π on the $[0, 2\pi)$ interval.

6. Give the most general solutions to the equation.

$$2\sin x\cos x - \sin(2x)\cos(2x) = 0$$

Part I: Simplify the left side of the expression so that each term involves only 2x. Factor the left side so that it can be solved. **(4 points)**

$2\sin x\cos x = \sin 2x$

$\sin 2x - \sin 2x\cos 2x = 0$

$\sin 2x(1 - \cos 2x) = 0$

Part II: Solve the factored equation. **(6 points)**

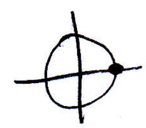
$\sin 2x(1 - \cos 2x) = 0$

$\sin 2x = 0$ or $1 - \cos 2x = 0$

$-\cos 2x = -1$

$\cos 2x = 1$

$\theta = 2x$



$\theta = 2x$

$\sin\theta = 0$

$\cos\theta = 1$

$\theta = 0$

$x = 0$

$\theta = \cancel{\theta}$,

$x = 0$

$$\boxed{x = 0}$$

7. Given the following identity, $\sec x\csc x(\tan x + \cot x) = 2 + \tan^2 x + \cot^2 x$:

Prove the identity by completing the table below, indicating the steps on the left and the reasoning on the right. **(12 points)**

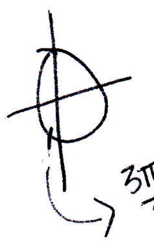
Calculation	**Reason**
$\sec x\csc x(\tan x + \cot x)$	Given in the problem
$\frac{1}{\cos x}\frac{1}{\sin x}\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$	Def. of $\sec\theta$, $\csc\theta$, $\tan\theta$, + $\cot\theta$
$\left(\frac{\sin}{\cos x\cos x\sin x} + \frac{\cos}{\cos x\sin x\sin x}\right)$	Distribute

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$	Simplify
$\sec^2 x + \csc^2 x$	Definition of secant and cosecant
$(1+\tan^2 x) + (1+\cot^2 x)$	Pythagorean identities; substitution
$2+\tan^2 x + \cot^2 x$	Simplify the expressions

8. Use the sum identity for tangent to find the *exact* value of $\tan\left(\frac{17\pi}{12}\right)$. No credit will be awarded for decimal approximations. **(6 points)**

$$\frac{\sin\frac{17\pi}{12}}{\cos\frac{17\pi}{12}} = \frac{\sin\frac{8.5\pi}{6}}{\cos\frac{8.5\pi}{6}} = \frac{\sin\frac{3}{2}\pi - \frac{1}{2}\sin\frac{\pi}{6}}{\cos\frac{3}{2}\pi - \frac{1}{2}\cos\frac{\pi}{6}} = \frac{\left(-1-\frac{1}{2}\left(\frac{1}{2}\right)\right)}{\left(0-\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right)}$$

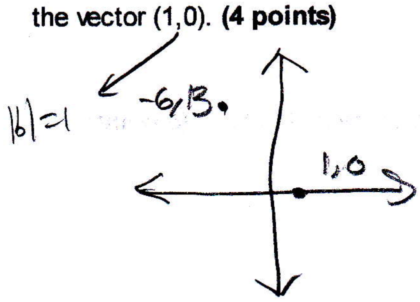
$$= \frac{-\frac{5}{4}}{-\frac{\sqrt{3}}{4}} = \boxed{\frac{-5}{-\sqrt{3}}}$$



$\frac{3\pi}{2} - \frac{\pi}{12}$ or $\frac{3\pi}{2} - \frac{1}{2}\left(\frac{\pi}{6}\right)$

9. Consider the vector $\mathbf{v} = (-6,13)$.

Part I: Find the angle (in degrees) between $\mathbf{v} = (-6,13)$ and the x-axis, which can be approximated as the vector (1,0). **(4 points)**



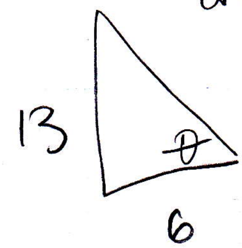
$$\theta = \frac{ab}{|a||b|} = \frac{(-6\cdot 1)+(13\cdot 0)}{|a||b|} = \frac{-6}{|a||b|}$$

$$|a| = \sqrt{-6^2+13^2} = \sqrt{36+169} = \sqrt{205}$$

$$= \frac{-6}{\sqrt{205}}$$

Part II: Write $\mathbf{v}$ in the form $(|\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta)$. Express the angle θ in degrees. **(4 points)**

$\arctan \frac{13}{6} = \theta$



$65.22° = \theta$

$V = 14.32 \cos 65.22 + 14.33 \sin 65.22$

$|v| = \sqrt{13^2+6^2} = 14.32$

Part III: Use the dot product of the vectors $\mathbf{v} = \langle -6, 13\rangle$ and $\mathbf{w} = \langle -42, -34\rangle$ to determine if they are orthogonal. **(2 points)**

v + w are not orthogonal

$(-6)(-42) + (13)(-34)$

$w = 252 + {}^{-}442$

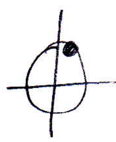
$w = -190$

$w \neq 0$

10. Find the fourth roots of the complex number $z_1 = 1 + \sqrt{3} \cdot i$.

Part I: Write z_1 in polar form. **(2 points)**

$1 + i\sqrt{3}$



$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$= 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

Part II: Apply De Moivre's theorem to find the modulus and angles of the roots of z_1. **(6 points)**

mod: 2

argument = angles = $\arctan\left(\frac{\sin \pi/3}{\cos \pi/3}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

Part III: What are the fourth roots of $z_1 = \sqrt{3} + i$? **(4 points)**

$\sqrt[4]{2}$

rt_1 $\sqrt[4]{2}$: $\sqrt[4]{2}\left(\cos(\sqrt{3}+0)+i\sin(\sqrt{3}+0)\right)$

$= \sqrt[4]{2}(\cos\sqrt{3}+i\sin\sqrt{3})$

$= \boxed{.19 + 1.17i}$

$rt_2 = \sqrt[4]{2}\left(\cos(\sqrt{3}+2\pi)+i\sin(\sqrt{3}+2\pi)\right)$

$\boxed{-1.18 + .11i}$

$rt_3 = \sqrt[4]{2}\left(\cos(\sqrt{3}+4\pi)+i\sin(\sqrt{3}+4\pi)\right)$

$\boxed{-.83 - .86i}$

$rt_4 = \sqrt[4]{2}\left(\cos(\sqrt{3}+6\pi)+i\sin(\sqrt{3}+6\pi)\right)$

$\boxed{-.19 + 1.17i}$