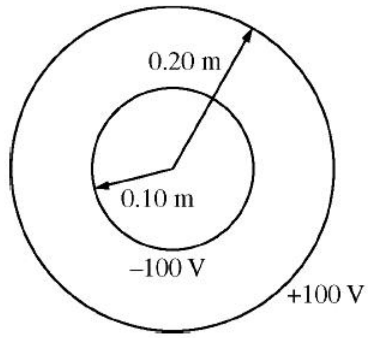
PHY132 Review Problems

For Exam 1

(Ch. 21-23 Giancoli)



1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of −100 V, and the outer shell is set at an electric potential of +100 V, with each potential defined relative to the conventional reference point. Let Q_i and Q_o represent the net charge on the inner and outer shells, respectively, and let r be the radial distance from the center of the shells. Express all algebraic answers in terms of Q_i, Q_o, r, and fundamental constants, as appropriate.

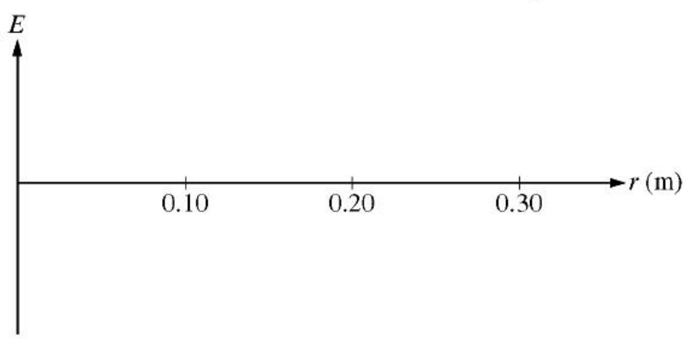
(a) Using Gauss's Law, derive an algebraic expression for the electric field $E(r)$ for $0.10 \text{ m} < r < 0.20 \text{ m}$.

(b) Determine an algebraic expression for the electric field $E(r)$ for $r > 0.20 \text{ m}$.

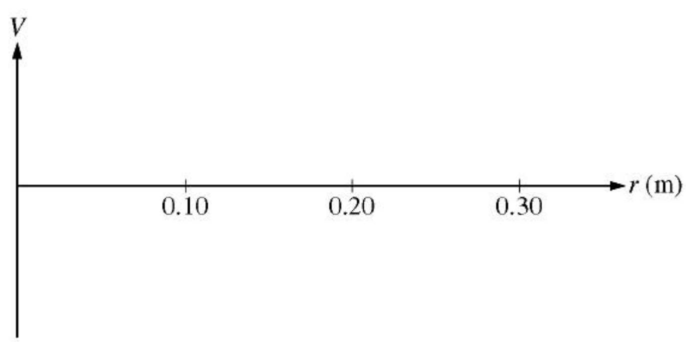
(c) Determine an algebraic expression for the electric potential $V(r)$ for $r > 0.20 \text{ m}$.

(d) Using the numerical information given, calculate the value of the total charge Q_T on the two spherical shells $(Q_T = Q_i + Q_o)$.

(e) On the axes below, sketch the electric field E as a function of r. Let the positive direction be radially outward.



(f) On the axes below, sketch the electric potential V as a function of r.



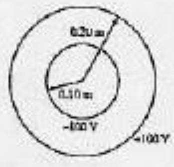
a) Using Gauss' Law, derive an expression for the electric field for 0.10 m < r < 0.20 m.

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_o}$$

$$E\oint dA = \frac{Q_i}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{Q_i}{\varepsilon_o}$$

$$E = \frac{Q_i}{4\pi\varepsilon_o r^2}$$



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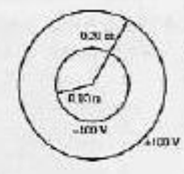
b) Determine E(r) for r > 0.20 m.

$$E = \frac{Q_{enc}}{4\pi\varepsilon_o r^2}$$

$$E = \frac{Q_i + Q_o}{4\pi\varepsilon_o r^2}$$



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c) Determine V(r) for r > 0.20 m.

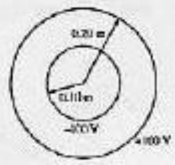
$$V = -\int E \cdot dr$$

$$V = -\int \frac{(Q_i + Q_o)}{4\pi\varepsilon_o r^2} dr$$

$$V = -\left(\frac{Q_i + Q_o}{4\pi\varepsilon_o}\right)\int \frac{1}{r^2} dr$$

$$V = -\left(\frac{Q_i + Q_o}{4\pi\varepsilon_o}\right)\left(-\frac{1}{r}\right)$$

$$V = \frac{Q_i + Q_o}{4\pi\varepsilon_o r}$$



Note: the full derivation was probably not necessary.

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d) Calculate the total charge $Q_T = Q_i + Q_o$.

At r = 0.20 m:

$$V = \frac{Q_i + Q_o}{4\pi\varepsilon_o r}$$

$$Q_T = Q_i + Q_o = 4\pi\varepsilon_o rV$$

$$Q_T = (4\pi\varepsilon_o)(0.20m)(+100V)$$

$$Q_T = 2.2\times10^{-9}C = 2.2nC$$

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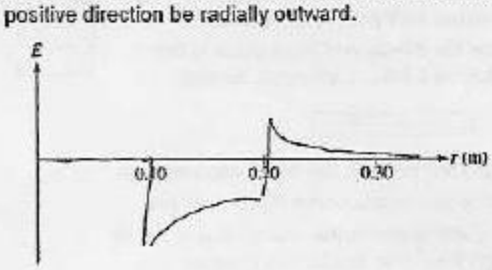
e) Sketch the electric field E as a function of r. Let the positive direction be radially outward.



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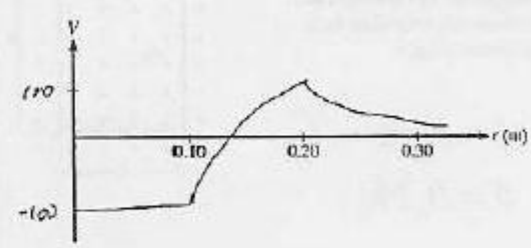
f) Sketch the electric potential V as a function of r.



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2. (15 points)

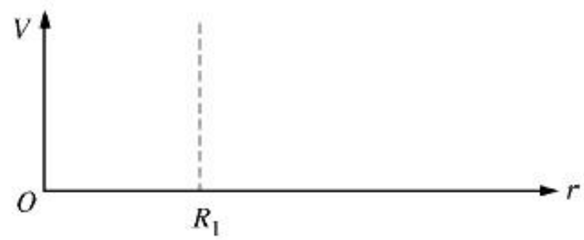
An isolated, solid copper sphere of radius $R_1 = 0.12\text{ m}$ has a positive charge of $6.4 \times 10^{-9}\text{ C}$.

(a)

i. Calculate the electric potential at a point 0.10 m from the center of the sphere.

ii. Calculate the electric potential at a point 0.24 m from the center of the sphere.

(b) On the axes below, sketch a graph of electric potential V versus radius r from the center of the sphere. Label the value at $r = 0$ on the vertical axis.

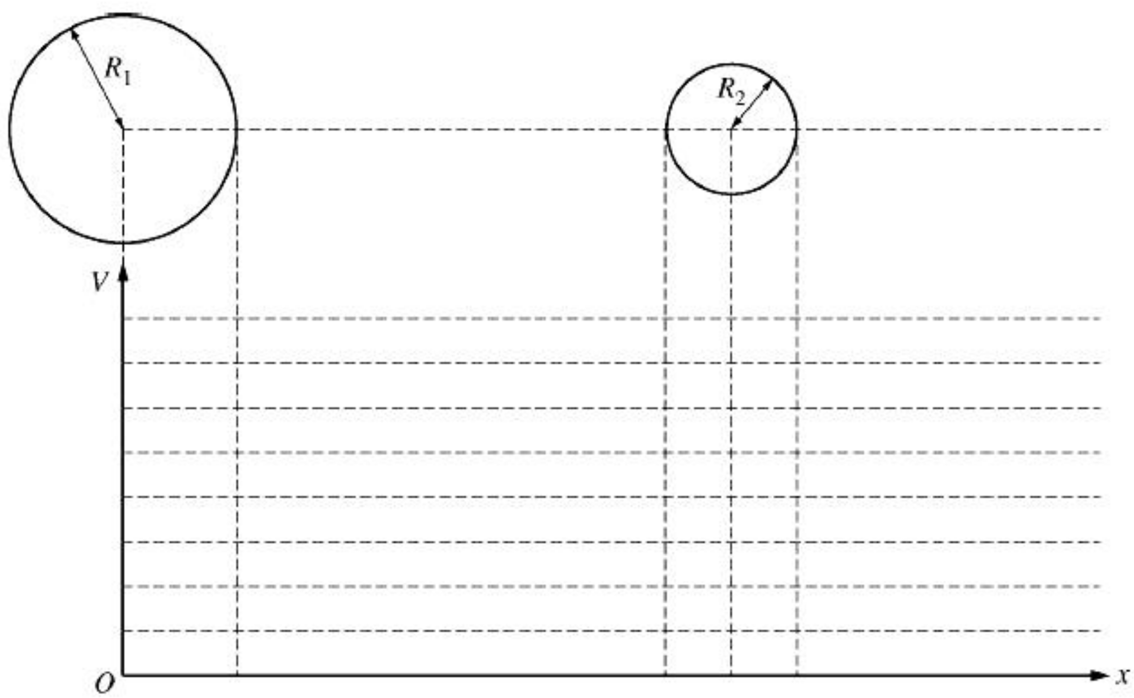


(c)

i. Determine the magnitude of the electric field at a point 0.10 m from the center of the sphere.

ii. Determine the magnitude of the electric field at a point 0.24 m from the center of the sphere.

(d) A second copper sphere of radius R_2 that is uncharged is placed near the first sphere, as represented in the figure below. On the axes below, sketch a graph of electric potential V versus distance along the x-axis shown, where the center of the first sphere is at $x = 0$.



(a)

i. 3 points

For writing the appropriate expression for the electric potential, given the spherical symmetry of the situation — 1 point

$$V = \frac{kq}{r}$$

The electric field inside a conducting sphere in equilibrium is zero, so the potential inside the sphere is constant, uniform, and equal to the potential at the surface.

$$V = \frac{kq}{R_1}$$

For correct substitutions — 1 point

$$V = \frac{\left(9.0 \times 10^9 \text{ N}\bullet\text{m}^2/\text{C}^2\right)\left(6.4 \times 10^{-9} \text{ C}\right)}{0.12 \text{ m}}$$

For the correct answer — 1 point

$V = 480 \text{ V}$

ii. 2 points

For writing the appropriate expression for the electric potential — 1 point

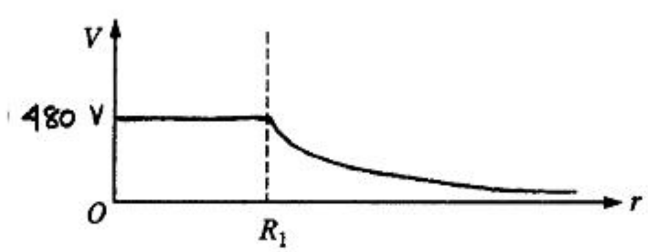
$$V = \frac{kq}{r}$$

$$V = \frac{\left(9.0 \times 10^9 \text{ N}\bullet\text{m}^2/\text{C}^2\right)\left(6.4 \times 10^{-9} \text{ C}\right)}{0.24 \text{ m}}$$

For the correct answer calculated from a correct expression showing substitutions — 1 point

$V = 240 \text{ V}$

(b) 3 points



For labeling the potential at $r = 0$ with the value from part (a) i — 1 point

For drawing a horizontal line from $r = 0$ to $r = R_1$ that is continuous with the other part of the graph — 1 point

For drawing a reasonable curve for the $1/r$ dependence in the region beyond R_1 — 1 point

(c)

i. 2 points

$r = 0.10 \text{ m} < 0.12 \text{ m} = R_1$

There is no electric field inside a conducting sphere in equilibrium.

For the correct answer 2 points

$E = 0 \text{ N/C}$

ii. 2 points

$r = 0.24 \text{ m} > 0.12 \text{ m} = R_1$, so the charge can be treated as a point charge at the center of the sphere.

For writing the appropriate expression for the electric field 1 point

$$E = \frac{kq}{r^2}$$

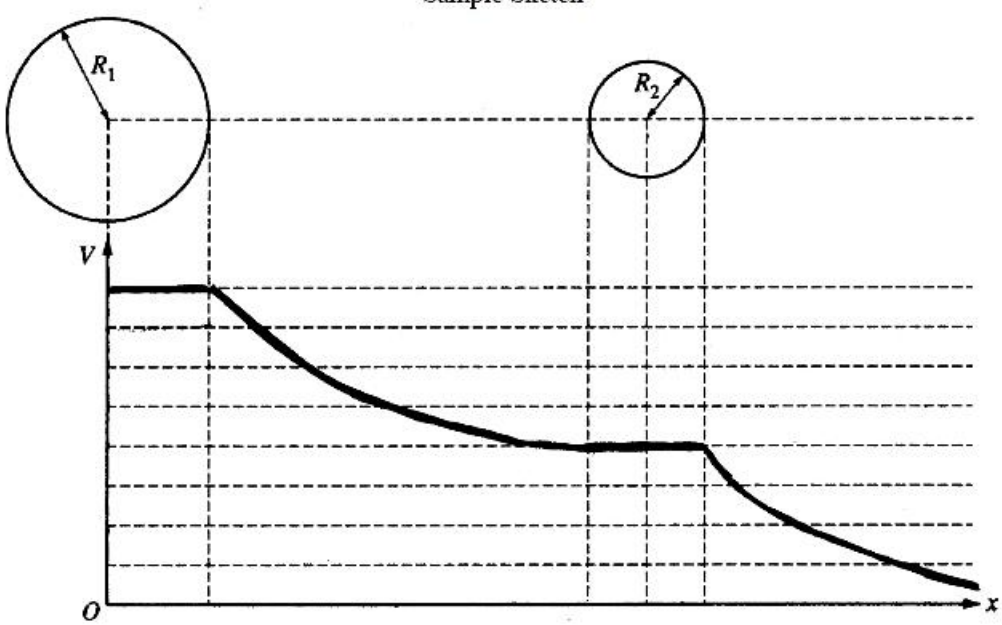
$$E = \frac{\left(9.0 \times 10^9 \text{ N}\bullet\text{m}^2/\text{C}^2\right)\left(6.4 \times 10^{-9} \text{ C}\right)}{(0.24 \text{ m})^2}$$

For the correct answer 1 point

$E = 1000 \text{ N/C}$

(d) 2 points

Sample Sketch



For a horizontal line inside the second sphere, with a nonzero value and continuous with the parts of the graph on either side 1 point

For $1/r$ curves in the regions on either side of the second sphere, with the curve in the region to the right at lower values than the curve to the left 1 point

Note: Except for the stipulation above for the $1/r$ curves, the relative values of the potentials of the two spheres are not scored. The sample graph shows an acceptable answer that does not include exact relative values.

Units point

For correct units in parts (a) ii and (c) ii 1 point

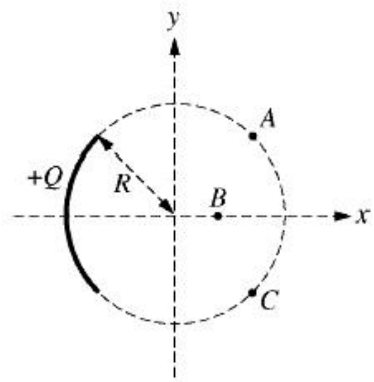


Figure I

3)

A charge $+Q$ is uniformly distributed over a quarter circle of radius R, as shown above. Points A, B, and C are located as shown, with A and C located symmetrically relative to the x-axis. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Rank the magnitude of the electric potential at points A, B, and C from greatest to least, with number 1 being greatest. If two points have the same potential, give them the same ranking.

____V_A ____V_B ____V_C

Justify your rankings.

Point P is at the origin, as shown below, and is the center of curvature of the charge distribution.

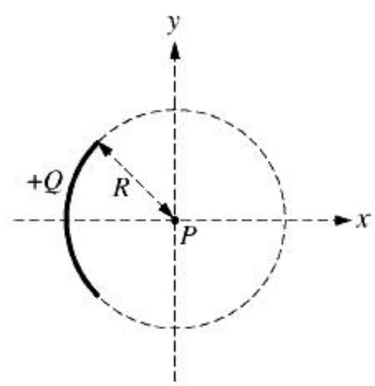
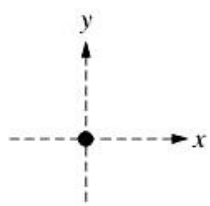


Figure II

(b) Determine an expression for the electric potential at point P due to the charge Q.

(c) A positive point charge q with mass m is placed at point P and released from rest. Derive an expression for the speed of the point charge when it is very far from the origin.

(d) On the dot representing point P below, indicate the direction of the electric field at point P due to the charge Q.



(e) Derive an expression for the magnitude of the electric field at point P.

(a) 3 points

For indicating that the potential at point B ranks 1 (has the highest potential) — 1 point
For indicating that the potentials at points A and C are equal and rank 2 — 1 point
For a correct justification — 1 point
Example: Compared to points A and C, point B is closer to most, and possibly all, points along the charge distribution. Since potential varies inversely with distance, point B has the highest potential. Points A and C have the same potential by symmetry.

(b) 2 points

For any indication of correct qualitative reasoning about the potential for this particular geometry — 1 point
Example: All points on the arc are a distance R from point P. Since potential is a scalar quantity, the potential will be the same as that of a point charge with charge Q located a distance R away.
For a correct answer — 1 point
$V = kQ/R$

Alternate solution — *Alternate points*
For indicating the potential is obtained by integrating the contributions from each part of the charge distribution — *1 point*

$$V = \int \frac{kdq}{r} \quad \text{where} \quad dq = \lambda r\, d\theta = \frac{Q}{r(\pi/2)} r\, d\theta = \frac{2Q}{\pi} d\theta$$

Noting that $r = R$ for the entire distribution, the integral becomes:

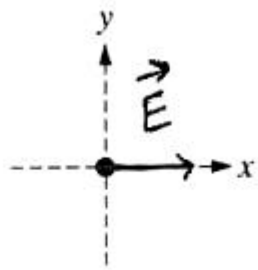
$$V = \frac{2kQ}{\pi R} \int_0^{\pi/2} d\theta$$

For a correct answer — *1 point*
$V = kQ/R$

(c) 4 points

For an indication that mechanical energy is conserved — 1 point
$U_i + K_i = U_f + K_f$
For correct substitution of potential energies — 1 point
$U_f = 0$
$U_i = qV$
For substituting the potential at P from part (b) — 1 point
$q(kQ/R) = K_f$
For substituting correctly for the kinetic energy and solving for the velocity — 1 point
$q(kQ/R) = (1/2)mv^2$
$v = \sqrt{2kqQ/mR}$

(d) 1 point



For a vector drawn in the correct direction — horizontally to the right 1 point

(e) 5 points

For any indication that the net electric field is the integral of all of the horizontal components from each part of the charge 1 point

$|E| = E_x = \int dE_x$

For correctly using cosine in computing the x-component 1 point

$|E| = \int dE\cos\theta = \int \frac{k\,dQ}{R^2}\cos\theta$

For changing variables to integrate with respect to θ 1 point

$dq = \frac{2Q}{\pi}d\theta$

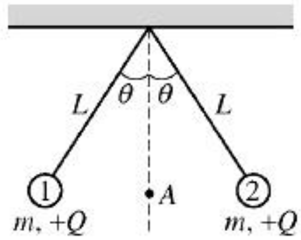
For correct limits of integration 1 point

$|E| = \int_{-\pi/4}^{\pi/4} \frac{2kQ}{\pi R^2}\cos\theta\,d\theta$ (or equivalent limits such as $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$)

$|E| = \frac{2kQ}{\pi R^2}\sin\theta\Big|_{-\pi/4}^{\pi/4}$

For a correct answer 1 point

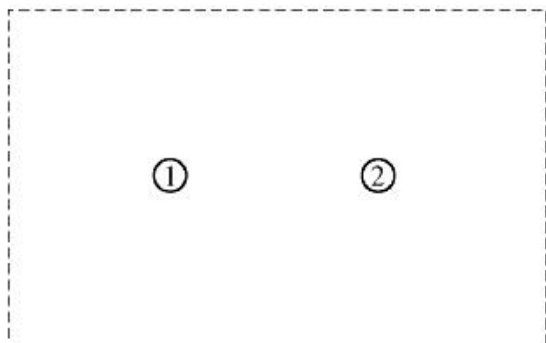
$|E| = \frac{2\sqrt{2}\,kQ}{\pi R^2}$



4) (10 points)

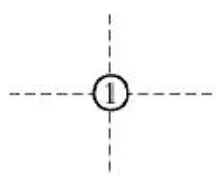
Two small objects, labeled 1 and 2 in the diagram above, are suspended in equilibrium from strings of length L. Each object has mass m and charge $+Q$. Assume that the strings have negligible mass and are insulating and electrically neutral. Express all algebraic answers in terms of m, L, Q, θ, and fundamental constants.

(a) On the following diagram, sketch lines to illustrate a 2-dimensional view of the net electric field due to the two objects in the region enclosed by the dashed lines.



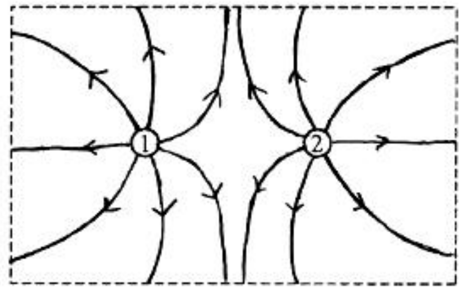
(b) Derive an expression for the electric potential at point A, shown in the diagram at the top of the page, which is midway between the charged objects.

(c) On the following diagram of object 1, draw and label vectors to represent the forces on the object.



(d) Using the conditions of equilibrium, write—but do not solve—two equations that could, together, be solved for θ and the tension T in the left-hand string.

(a) 3 points



For the direction of all field lines away from the source charges — 1 point

For the correct shape, symmetry, and curvature of the field lines — 1 point

For a clear indication that the net field about point *A*, the point midway between the two charges, is zero; example: an absence of field lines in the area around point *A* — 1 point

(b) 2 points

For clearly showing the addition of both contributions to the potential at point *A* — 1 point

$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$ OR $V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{d} + \frac{Q}{d}\right)$ OR equivalent

For a correct expression in terms of the given quantities — 1 point

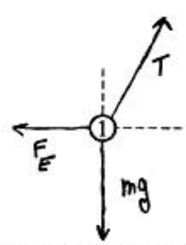
$V = \frac{2kQ}{L\sin\theta}$ OR $\frac{2Q}{4\pi\epsilon_0 L\sin\theta}$ OR $\frac{Q}{2\pi\epsilon_0 L\sin\theta}$ OR equivalent

Notes:

- The answer with no work shown earned 1 point.
- The following expression or its equivalents, with no other work shown, earned both points:

$$V = \frac{kQ}{L\sin\theta} + \frac{kQ}{L\sin\theta}$$

(c) 2 points



For all three vectors correctly drawn, with arrowheads, and no extraneous vectors — 1 point

For appropriate labeling of vectors (only if first point was awarded) — 1 point

(d) 3 points

For an expression indicating that the *x*-component of tension is equal to F_E — 1 point

$T_x - F_E = 0$ OR $T\sin\theta = F_E$ OR equivalent

For an expression indicating that the *y*-component of tension is equal to *mg* — 1 point

$T_y - mg = 0$ OR $T\cos\theta = mg$ OR equivalent

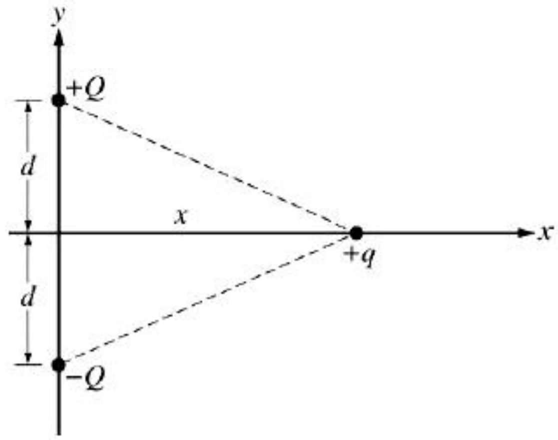
For stating the two equations in terms of the given quantities — 1 point

$T\sin\theta = \frac{kQ^2}{4L^2\sin^2\theta}$ OR $\frac{Q^2}{16\pi\epsilon_0 L^2\sin^2\theta}$ OR $\frac{1}{4\pi\epsilon_0}\frac{Q^2}{4L^2\sin^2\theta}$ OR equivalent

$T\cos\theta = mg$

Notes:

- Correct statement of both of the final two equations, in the absence of any other expressions, earned full credit.
- One point was deducted if sine and cosine were interchanged at any point.



5) (15 points)

Three electric charges are arranged on an x-y coordinate system, as shown above. Express all algebraic answers to the following parts in terms of Q, q, x, d, and fundamental constants.

(a) On the diagram, draw vectors representing the forces F_1 and F_2 exerted on the $+q$ charge by the $+Q$ and $-Q$ charges, respectively.

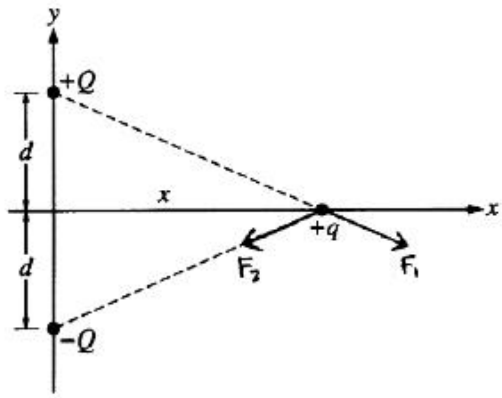
(b) Determine the magnitude and direction of the total electric force on the $+q$ charge.

(c) Determine the electric field (magnitude and direction) at the position of the $+q$ charge due to the other two charges.

(d) Calculate the electric potential at the position of the $+q$ charge due to the other two charges.

(e) Charge $+q$ is now moved along the positive x-axis to a very large distance from the other two charges. The magnitude of the force on the $+q$ charge at this large distance now varies as $1/x^3$. Explain why this happens.

(a) 2 points



For indicating the correct direction for the force due to the $+Q$ charge (F_1 as drawn above) 1 point

For indicating the correct direction for the force due to the $-Q$ charge (F_2 as drawn above) 1 point

(b) 6 points

For any indication that the magnitudes of F_1 and F_2 are the same 1 point

The x-components of F_1 and F_2 cancel.

For any indication that the magnitude of the net force is the sum of the y-components of F_1 and F_2, which are equal 1 point

Example: $F_{total} = F_1 \cos\theta + F_2 \cos\theta = 2F\cos\theta$, where θ is the angle between the y-axis and the dashed lines in the figure

For a correct expression for $\cos\theta$ 1 point

$$\cos\theta = \frac{d}{\sqrt{x^2 + d^2}}$$

For a correct substitution for F into the above expression for F_{total} 1 point

$$F = \frac{kqQ}{r^2} = \frac{kqQ}{x^2 + d^2}$$

$$F_{total} = 2\frac{kqQ}{x^2 + d^2}\frac{d}{\sqrt{x^2 + d^2}}$$

For the correct magnitude of the total force 1 point

$$F_{total} = \frac{2kqQd}{\left(x^2 + d^2\right)^{3/2}} \quad \text{or equivalent}$$

For indicating the correct direction for the total force, e.g., negative y-direction, toward the bottom of the page, etc. 1 point

(c) 2 points

The field can be found from the force.

$E = F_{total}/q$

For the correct magnitude of the electric field — 1 point

$$E = \frac{2kQd}{\left(x^2 + d^2\right)^{3/2}}$$

For indicating the correct direction for the electric field, e.g., negative y-direction, toward the bottom of the page, etc. — 1 point

(d) 2 points

The total potential is the sum of the individual point charge potentials.

$$V = V_1 + V_2 = \frac{kQ}{\sqrt{x^2 + d^2}} + \frac{-kQ}{\sqrt{x^2 + d^2}}$$

For indicating that the electric potential is zero — 2 points

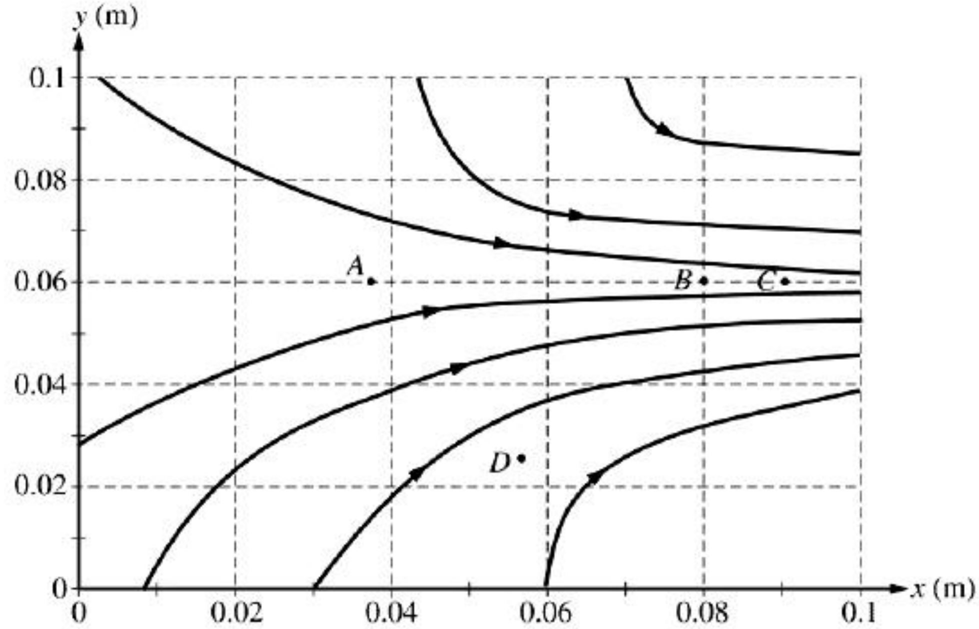
Note: One point partial credit could be earned for only recognizing that the potentials from the two charges must be added.

(e) 3 points

For any indication that as x gets large, the hypotenuse and x are approximately equal or d is negligible compared to x — 1 point

For indicating that the above implies that $\sqrt{x^2 + d^2} \approx x$ — 1 point

For indicating that substituting the approximate equality into the answer from part (b) yields $F_{total} = \dfrac{2kqQd}{x^3}$ — 1 point



6)

Consider the electric field diagram above.

(a) Points *A*, *B*, and *C* are all located at $y = 0.06$ m .

i. At which of these three points is the magnitude of the electric field the greatest? Justify your answer.

ii. At which of these three points is the electric potential the greatest? Justify your answer.

(b) An electron is released from rest at point *B*.

i. Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.

ii. Calculate the electron's speed after it has moved through a potential difference of 10 V.

(c) Points *B* and *C* are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.

(d) On the diagram, draw an equipotential line that passes through point *D* and intersects at least three electric field lines.

(a)

(i) 2 points

For indicating that the electric field magnitude is greatest at point C — 1 point
For a correct justification — 1 point
For example: Field lines are drawn closer together where the field is greater.
Note: No credit was awarded for the justification if an incorrect point was chosen.

(ii) 2 points

For indicating that the electric potential is greatest at point A — 1 point
For a correct justification — 1 point
For example: The field along $y = 0.6$ m is toward the right. The field points in the direction of decreasing potential, so A must be at the highest potential.
Note: No credit was awarded for the justification if an incorrect point was chosen.

(b)

(i) 4 points

For indicating that the electron moves to the left, stated explicitly or implied — 1 point
For indicating that the speed increases — 1 point
For indicating that the acceleration is directed to the left, stated explicitly or implied — 1 point
For indicating that the magnitude of the acceleration decreases — 1 point
Example of a good answer: The force on an electron is opposite to the field, so it will move left. The field is weaker to the left so the acceleration will decrease. As long as there is a force on the electron, its speed will continue to increase to the left.

(ii) 3 points

For using conservation of energy with $U = qV$ — 1 point

$$\frac{1}{2}mv^2 = q\,\Delta V$$

$$v = \sqrt{2q\,\Delta V/m}$$

For correct substitution of values into either equation above — 1 point

$$v = \sqrt{2\left(1.6 \times 10^{-19}\ \text{C}\right)(10\ \text{V})/\left(9.11 \times 10^{-31}\ \text{kg}\right)}$$

For the correct answer — 1 point

$$v = 1.9 \times 10^{6}\ \text{m/s}$$

Note: Substitution point was awarded if correct answer was indicated.

(c) 2 points

$$E = -\frac{\Delta V}{r}$$

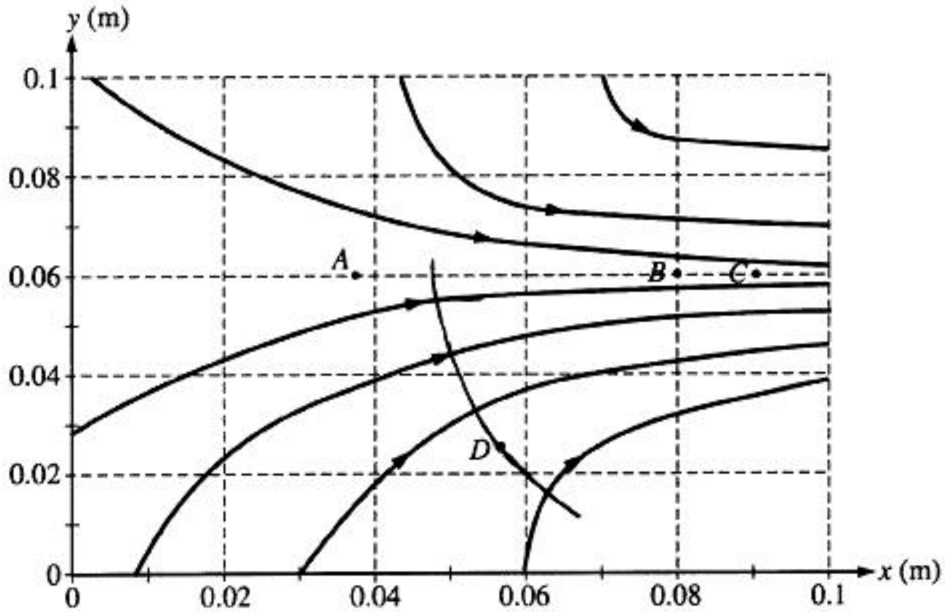
$$E = \frac{20\ \text{V}}{0.01\ \text{m}}$$

For the correct answer with correct units — 1 point

$E = 2000\ \text{V/m}$ or $2000\ \text{N/C}$

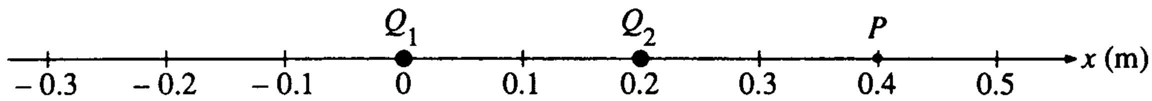
For the correct assumption that the field is close enough to uniform in this region to do a calculation as if it were — 1 point

(d) 2 points



For drawing a curved line concave up or concave right that passes through point D and at least three electric field lines — 1 point

For drawing the curved line perpendicular to at least three field lines — 1 point



7)

Two point charges, Q_1 and Q_2, are located a distance 0.20 meter apart, as shown above. Charge $Q_1 = +8.0\mu C$. The net electric field is zero at point *P*, located 0.40 meter from Q_1 and 0.20 meter from Q_2.

(a) Determine the magnitude and sign of charge Q_2.

(b) Determine the magnitude and direction of the net force on charge Q_1

(c) Calculate the electrostatic potential energy of the system.

(d) Determine the coordinate of the point *R* on the *x*-axis between the two charges at which the electric potential is zero.

(e) How much work is needed to bring an electron from infinity to point *R*. which was determined in the previous part?

(a) 4 points

For equation for electric field: $E = \frac{kQ}{r^2}$ 1 point

For recognition of superposition at point P: $\mathbf{E}_1 + \mathbf{E}_2 = 0$ 1 point

$$K\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right) = 0$$

$$Q_2 = -Q_1 \frac{r_2^2}{r_1^2}$$

For negative sign in above equation, or some indication that $\mathbf{E}_1$ and $\mathbf{E}_2$ are in opposite directions 1 point

For correct calculation $Q_2 = -(8.0\ \mu C)\ \frac{(0.2\ m)^2}{(0.4\ m)^2} = -2\ \mu C$ 1 point

(3 of the 4 points were awarded for correct use of the erroneous equation $E = \frac{kQ}{r}$)

(b) 3 points

$F = \frac{kQ_1Q_2}{r^2}$ or $F = QE$ — 1 point

For correct calculation: $F = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8\ \mu\text{C})(2\ \mu\text{C})}{(0.2\ \text{m})^2}$

$= 3.6\ \text{N}$ — 1 point

For correct direction: to the right — 1 point

Also acceptable were "attractive" or "zero degrees" (i.e., the angle from the x-axis in an x-y coordinate system)

The points for the calculation and the direction were each awarded only if they were consistent with answer to part (a).

(c) 3 points

$U = \frac{kQ_1Q_2}{r}$ — 1 point

For correct calculation: $U = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8\ \mu\text{C})(-2\ \mu\text{C})}{(0.2\ \text{m})}$

$= -\ 0.72\ \text{J}$ — 1 point

For negative sign in above answer — 1 point

(d) 3 points

$V = \frac{kQ}{r}$ — 1 point

For any statement of $V_1 + V_2 = 0$ — 1 point

$$\frac{kQ_1}{x} + \frac{kQ_2}{0.2\ \text{m} - x} = 0$$

$$\frac{Q_1}{x} = \frac{-Q_2}{0.2\ \text{m} - x}$$

$$(0.2\ \text{m} - x)Q_1 = -xQ_2$$

$$x = \frac{(0.2\ \text{m})Q_1}{Q_1 - Q_2}$$

For correct calculation: $x = \frac{(0.2\ \text{m})(8\ \mu\text{C})}{8\ \mu\text{C} - (-2\ \mu\text{C})} = 0.16\ \text{m}$ — 1 point

(e) 1 point

Work = 0 because $V = 0$ at point R — 1 point

Additional 1 point awarded for correct use of units and no incorrect units. — 1 point

If no units were used and the substitutions for values of charges did not include " $\times\ 10^{-6}$," 1 point was deducted.