For all of my proofs, I will be using the notation given in the assignment.

**Exercise 1:**

Preliminary: Proof that *u\** does not belong to *P*, or equivalently, that no path exists between *u* and *u\** in graph *G’*:

Given our supposition that Lemma 1 is false, we know by definition that the edge incident on vertex *u* can only exist in matching *M’* and not *M*. It follows that this edge must be in the set *A’*.

We also know *P* has the property of alternating between edges in *A* and *A’*. Therefore, in order to connect *u\** to *u*, we must first traverse an edge in *A’* towards *V*. To return, we must traverse an edge in *A* back to the set of vertices *U* and repeat the process until we end up on vertex *u*. However, we established that there is no edge in *A* that is incident on vertex *u* because vertex *u* is only matched in *M’*, and therefore this edge only exists in *A’*.

Therefore, *u\** is not in *P*.

**Completion of proof of Lemma 1 with a *P* of odd length:**

Given *P* of odd length, we know that |*A’*| > |*A*| since one end of *P* must begin on *u*.

If we consider matching *M*, we know the endpoints of path *P* are unmatched.

However, if we replace the edges represented by *A* in *M* with the edges represented by *A’*, we end up with a new matching of greater cardinality than *M* (i.e. |*M*| < |*M*|-|*A*|+|*A’*|).

Since we had originally stated that *M* was a MWMCM, there is a contradiction since there exists in *G* a matching containing *A’* instead of *A* that has a greater cardinality than *M*.

This proof holds even with the addition of *u\** since we proved before that *u\** is not in *P* and therefore cannot be in this matching that has a greater cardinality.

**Exercise 2:**

Retain the proof from Exercise 1 that *u\** is not in *P*.

**Completion of proof of Lemma 1 with a *P* of even length:**

Given *P* of even length, we can create alternate matchings for *M* or *M’* using *A* and *A’* like we did in Exercise 1. However, these new matchings where *A* and *A’* are swapped have equal cardinality as their original matchings, ruling out the simple proof that *M* is not a MWMCM.

Looking at Lemma 1, we see that we simply want to find any MWMCM where *u* is unmatched in both *M* and *M’*. In this case, it is a simple matter to see that if we replaced *A’* with *A* in *M’* similar to Exercise 1, we can arrive at a MCM where *u* is unmatched in both *M* and *M’*.

Given the knowledge that *u\** is not in *P*, we need to prove that this new matching is also maximum-weight. By replacing edges *A’* in *M’* with edges *A*, we arrive at a MCM (call it *M’’*) with equal weight as the original *M’*, which we know is maximum-weight. We know that *M’’*  will be of equal weight because neither set *A’* nor *A* contain *u\**, the only element that could have increased the weight of *M’* via its new incident edges, and by the fact that the original matching *M* contained the edges in *A* and was maximum-weight.

Therefore, we have successfully produced a MWMCM *M’’* where vertex *u* is unmatched, as in the original matching *M*. This contradicts the assumption that this matching does not exist in *G’*.