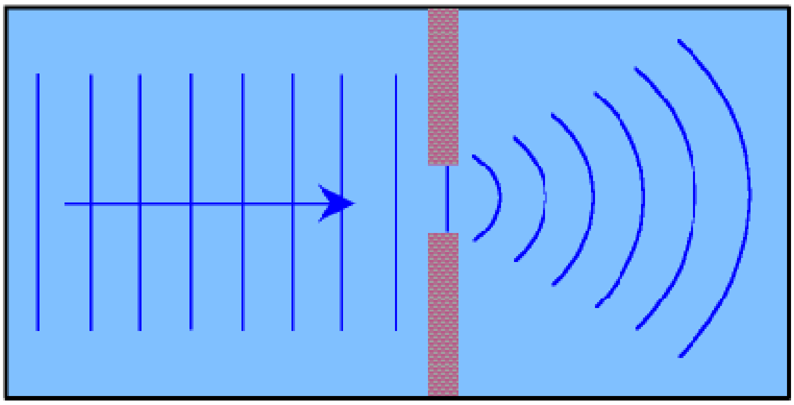
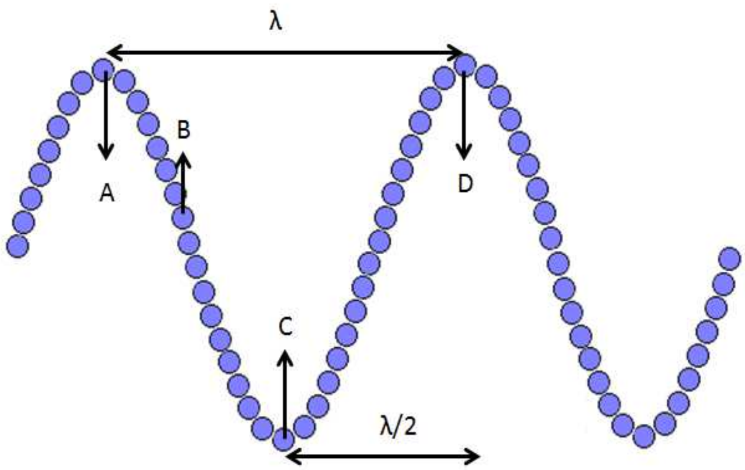


Wave Systems



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AS 3.3 Demonstrate understanding of wave systems (90520)

Objectives:

- **To give concise explanations that show clear a understanding of the following physics phenomena, concepts, principles and/or relationships:**

  - Interference of electromagnetic and sound waves
  - Multi-slit interference and diffraction gratings
  - Standing waves in strings and pipes
  - Resonance
  - Harmonics and overtones
  - Beats
  - The Doppler Effect (stationary observer)

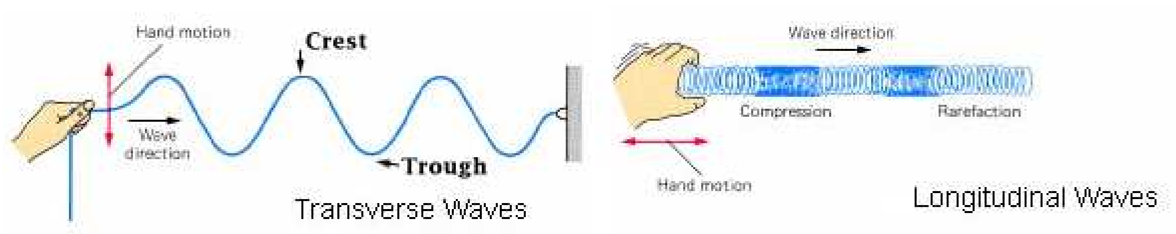
- **To solve complex problems using the following formulae:**

  - $f = \frac{1}{T}$
  - $v = f\lambda$
  - $n\lambda = \frac{dx}{L}$
  - $dsin(\theta) = n\lambda$
  - $f' = f\frac{v_w}{v_w \pm v_s}$

Section 1 - General wave properties

Waves are a means of carrying energy without transferring matter. There are two types of wave:

- **Transverse** waves = Particles vibrate *perpendicular* to the direction of wave propagation via crests and troughs e.g. *electromagnetic* waves which do not require a medium like air
- **Longitudinal** waves = Particles vibrate *parallel* to the direction of wave propagation via compressions and rarefactions e.g. *sound waves* which do require a medium like air



[1]

All waves use the following 5 measurements:

- **Amplitude (A)** = The maximum displacement of wave particles from equilibrium (e.g. half the vertical distance from crest to trough) measured in metres (m)
- **Wavelength (λ)** = The distance between two corresponding particles on a wave (e.g. horizontal distance from crest to crest) measured in metres (m)
- **Frequency (f)** = The number of waves per second, measured in Hertz (Hz or s^{-1})
- **Period (T)** = The time it takes for one complete wave, measured in seconds (s)

$$f = \frac{1}{T}$$

- **Speed (v)** = How fast a wave can travel, measured in metres per second (ms^{-1})

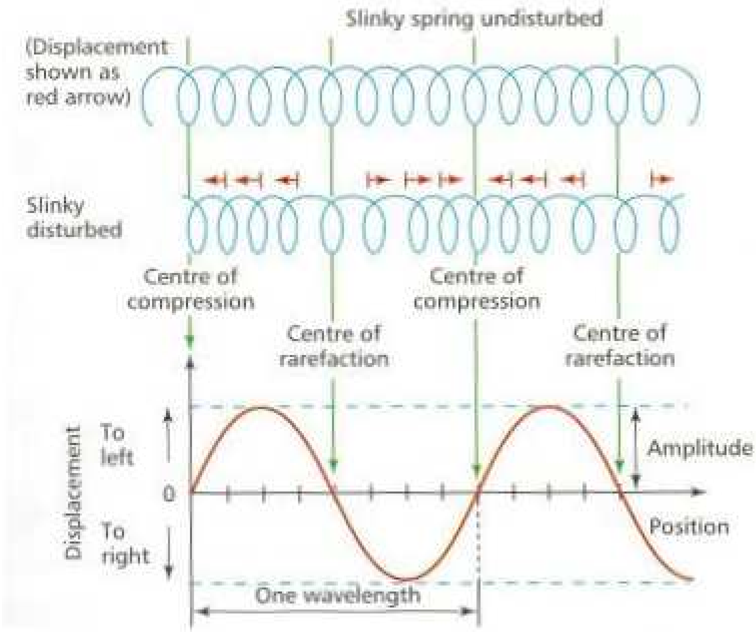
$$v = \frac{d}{t} = \frac{\lambda}{T} = f\lambda$$

Electromagnetic waves all travel at $c = v = 3.0 \times 10^8\ ms^{-1}$ in *vacuum* but their frequencies and wavelengths vary. The **electromagnetic spectrum** involves all electromagnetic waves. In order of **increasing wavelength** (and thus decreasing frequency and energy) this is:

Gamma rays < X-rays < Ultraviolet (UV) < Visible light (blue < red) < Infrared (IR) < Microwaves < Radio waves

Sound waves travel at approximately 330 ms^{-1} in air and travel **faster** in more dense mediums.

[1] Images sourced and edited from www.tutorvista.com/content/physics/physics-i/wave-motion-sound/longitudinal-and-transverse-waves.php



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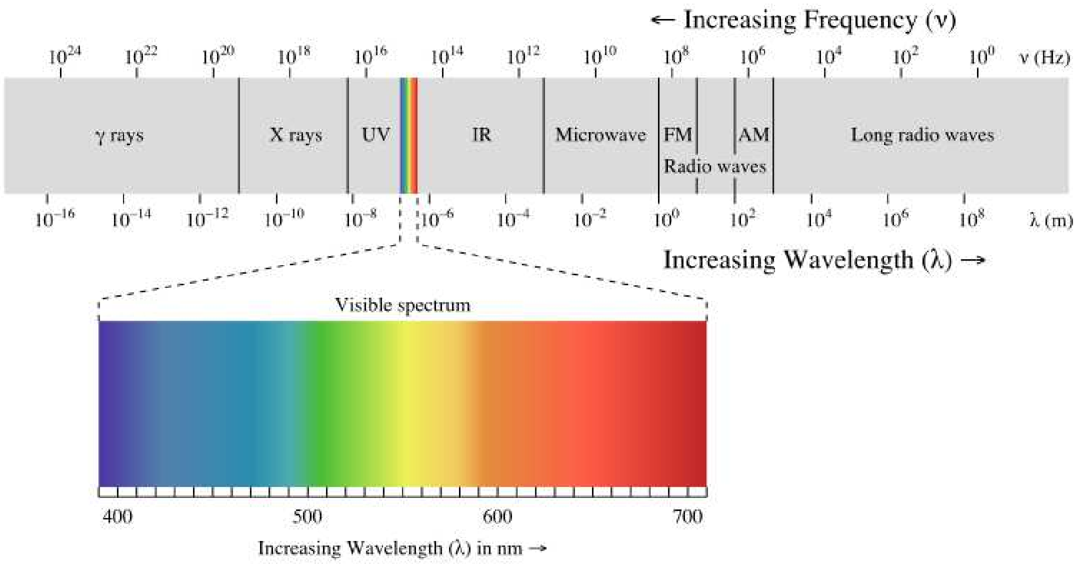


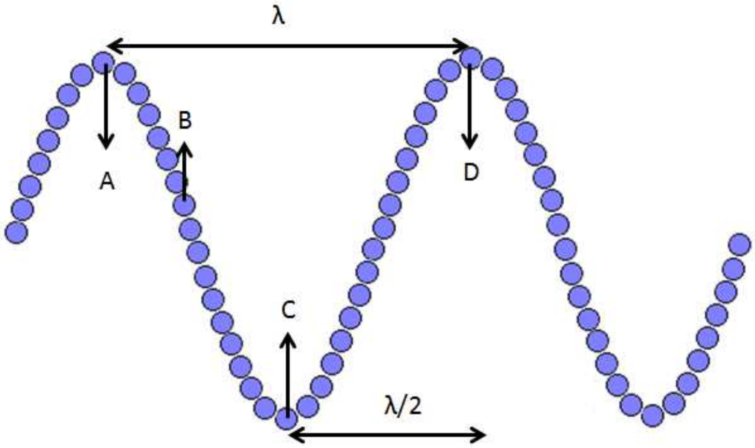
[3]

The term **phase** refers to the motion of two particles on the same wave. Two particles are said to be either:

- **In phase** if they are moving in the *same* direction at all times. These particles are **exactly in phase** (0°) if they are a whole number of wavelengths apart.
- **Out of phase** if they are moving in opposite directions at all times. These particles are **exactly out of phase** (180°) if they are an odd number of half wavelengths apart.

It is the *phase difference* of particles along a wave which results in the transfer of energy in the direction of wave propagation.

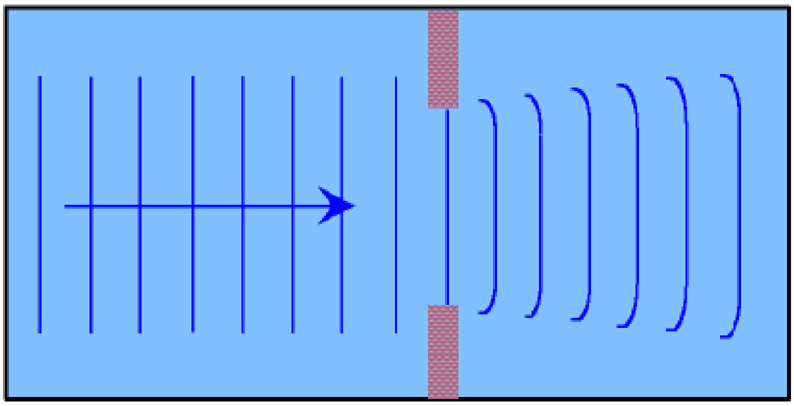
[2] Image sourced from http://teacherhaniza.blogspot.co.nz/2012/01/waves.html
[3] Image sourced from http://en.wikipedia.org/wiki/Electromagnetic_radiation

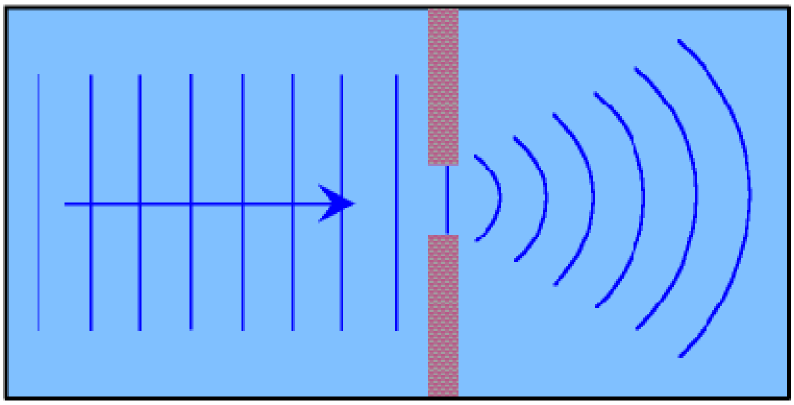


4

Diffraction refers to the bending of waves around a barrier or through a gap. During this process the wavelength, frequency and speed *do not change* but the direction of wave propagation changes and the wave fronts *change shape*.

This pattern is most evident when the wavelength is **greater than or equal to** the size of the gap.





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[4] Image sourced and edited from http://library.thinkquest.org/15433/unit5/5-1.htm
[5] Images sourced from www.smkbud4.edu.my/Data/sites/vschool/phy/wave/diffraction.htm

Section 1 – Questions

1. Light can travel at $3.0 \times 10^8\ ms^{-1}$ in vacuum. Red light has a wavelength of 650 nm and blue light has a frequency of $6.7 \times 10^{14}\ Hz$.

   i) Are light waves transverse or longitudinal? Explain your answer.

   Light waves can travel without a medium (in vacuum) and therefore are transverse. The direction of particle movement is perpendicular to the direction of wave propagation.

   ii) How far could a light wave travel in one microsecond?

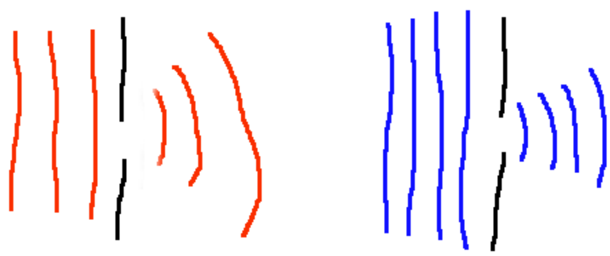
   $v = \frac{d}{t} \rightarrow d = vt = 3.0 \times 10^8\ ms^{-1} \times 1.0 \times 10^{-6}\ s = 300\ m$

   iii) Find the frequency of red light. Give your answer to the correct number of significant figures.

   $v = f\lambda \rightarrow f = \frac{v}{\lambda} = (3.0 \times 10^8\ ms^{-1}) \div (650 \times 10^{-9}\ m) = 4.6 \times 10^{14}\ Hz\ (2sf)$

   iv) Determine whether red or blue light would diffract to the greatest extent. Explain your answer and draw a diagram clearly showing the differences in the diffraction patterns of red and blue light through a small gap of the same size.

   Red light has a greater wavelength (lower frequency at the same speed) than blue light and will therefore diffract to a greater extent through a small gap of the same size.



2. Sound waves travel through air at 330 ms^{-1}. Answer the following questions based on the diagram of a sound wave below.

   i) What is the wavelength of this sound wave?

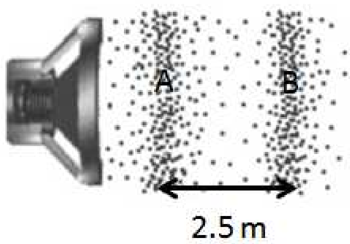
   $\lambda = 2.5\ m$

   ii) What is the period of this sound wave?

   $v = \frac{\lambda}{T} \rightarrow T = \frac{\lambda}{v} = 2.5\ m \div 330\ ms^{-1} = 7.6 \times 10^{-3}\ s$

   iii) What is the phase of particles at A and B? Explain your answer.

   Particles at A and B are both at the compression stages and are therefore both moving in the same direction at the same time. They are therefore exactly in phase (0°)



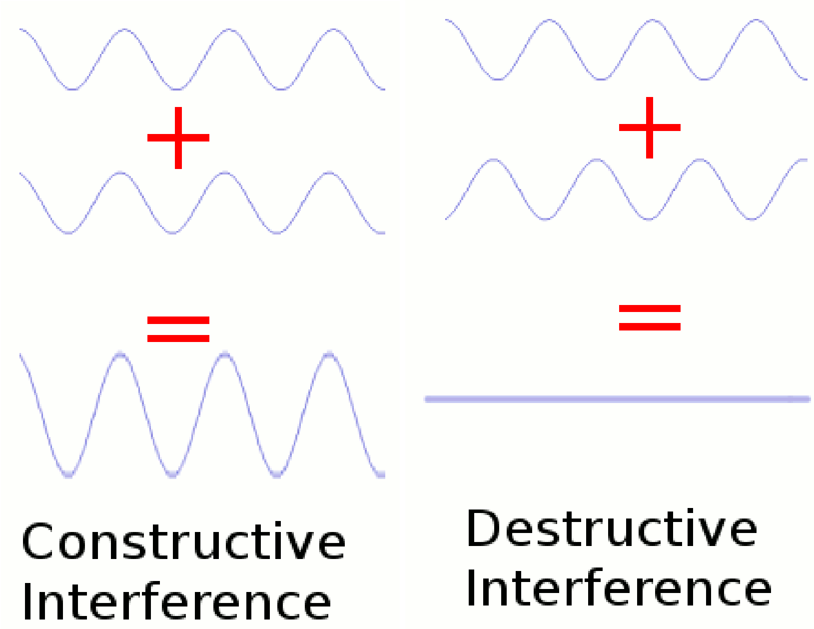
[6]

[6] Image sourced and edited from http://phonoscopy.com/SonicGeologic/SonicGeologic.html

Section 2 - Interference of Waves

When two waves of the *same frequency* pass each other, the displacements of their particles can combine in a process known as **interference**. This can be:

- **Constructive interference**, where the waves are *in phase* and the crests or troughs add together to give *increased* amplitude.
- **Destructive interference**, where the waves are *out of phase* and the crests and troughs cancel to give *decreased* amplitude.



[7]

A **point source** produces wave fronts which radiate outwards in a *circular fashion* at the **same speed**. If two or more point sources produce waves of *identical frequency and amplitude*, the waves from each source will interfere regularly, resulting in an **interference pattern**:

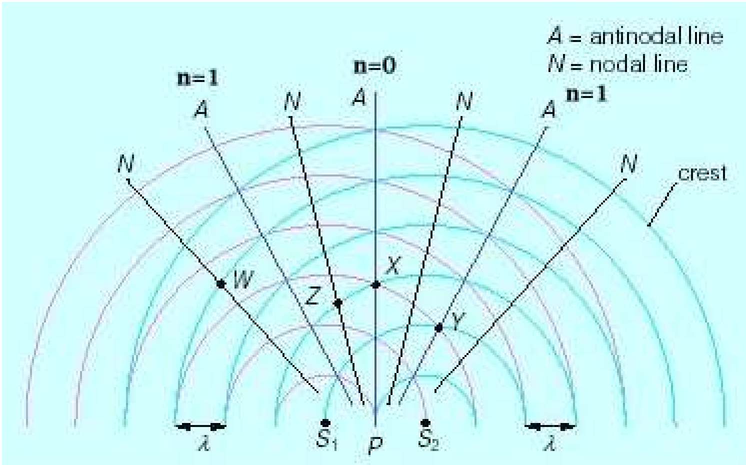
- **Antinodal lines** are lines of *constructive interference* with maximum amplitude. They are the result of waves from one point source being a *whole number of wavelengths* apart from (and exactly in phase with) waves from another point source.

$$\textit{Path difference} = n\lambda$$

- **Nodal lines** are lines of *destructive interference* with zero amplitude. They are the result of waves from one point source being an *odd number of half wavelengths* apart from (and exactly out of phase with) waves from another point source.

$$\textit{Path difference} = (n - \frac{1}{2})\lambda$$

[7] Sourced from http://ffden-2.phys.uaf.edu/212_spring2011.web.dir/michael_hirte/interference.htm



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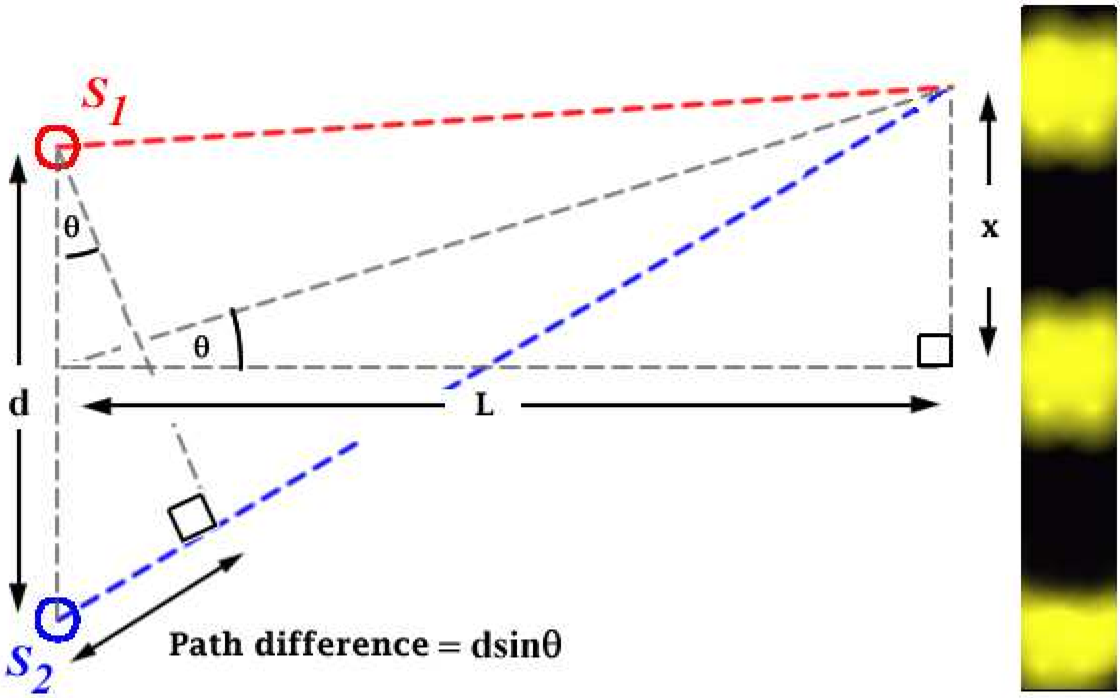
For sound waves, frequency reflects **pitch** and amplitude reflects **loudness**. Therefore **two point sources** which produce sound waves of *identical pitch and loudness* will set up an interference pattern. The antinodal lines will have *maximum loudness* while nodal lines will be *silent*.

For light waves, frequency reflects **colour** and amplitude reflects **brightness**. The **two point sources** must therefore produce light waves of *identical colour and brightness* in order to create an interference pattern. The light waves must also be **coherent** - this is achieved by shining a single beam of *monochromatic* light through two thin slits (as opposed to using two beams). On a screen, the antinodal lines will produce *bright fringes* while the nodal lines will produce *dark fringes*.



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8 Sourced and edited from http://sciencecity.oupchina.com.hk/npaw/student/glossary/nodal_line.htm
9 Sourced and edited from http://mrmaloney.com/mr_maloney/ap/docs/ch16+17-WavesSound/wave-notes.html

As one can see in Diagram 9, when two such point sources (**S_1** and **S_2**) of light are a distance of ***d*** *metres* apart, they can create an interference pattern of *bright and dark fringes* on a screen a distance of **L** metres away. The distance of any fringe from the central bright fringe (central antinodal line, n=0) is ***x*** *metres* while the angle between these two fringes and the halfway point between the two sources is **θ**.

If **x is small when compared to L** (for a small angle), then we can approximate:

$$tan(\theta) = \frac{x}{L} \approx sin(\theta)$$

The *path difference* resulting in the interference pattern is therefore **approximated** by:

$$Path\ difference\ =\ dsin(\theta) \approx \frac{dx}{L}$$

If we recall the equations for *path difference* of **nodal** and **antinodal** lines, we can then obtain the following two approximations:

$$For\ any\ antinodal\ line\ (bright\ fringe) \rightarrow\ n\lambda = \frac{dx}{L}$$
$$For\ any\ nodal\ line\ (dark\ fringe)\ \rightarrow\ (n - \frac{1}{2})\lambda = \frac{dx}{L}$$

Therefore, for *constant values of d, λ and L*, we can see that *n is proportional to x* so the fringes are **evenly spaced**. This spacing will *increase* (and therefore less fringes will be seen) with:

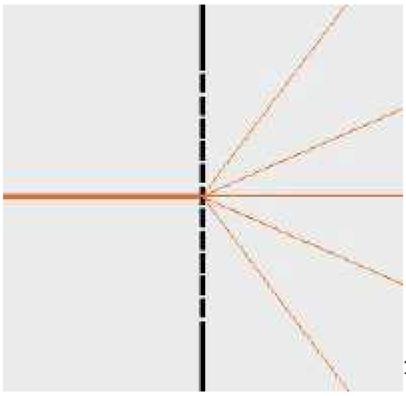
- **A smaller distance - d -** between light sources
- **A larger distance - L -** between the light sources and the screen
- **A larger wavelength - λ -** of light used (or therefore a smaller frequency)

If the spacing between fringes is **too small**, however, the bright fringes may *overlap* and result in *no interference pattern* being seen, only a *single bright patch*.

Now, if **multiple point sources** (instead of just two) produce waves of *identical frequency and amplitude*, then a similar interference pattern results.

For light, this pattern is often created by use of a **diffraction grating**. This is a piece of glass or plastic with many parallel slits cut into it. The *distance between the slits (d)* in metres can be related to the *number of slits per metre (N)* on the material by the following formula:

$$\boldsymbol{d = \frac{1}{N}}$$


[10]

In general, the interference pattern created from shining a single beam of monochromatic light through a *diffraction grating* is *similar* to that from shining the beam through *only two slits*. There are, however, several differences:

- The light diffracts at a **wider angle** (maximum 180°) because the slits are *very thin*
- The fringes are **more widely spread** because the point sources are *more closely spaced*
- The bright fringes are **more sharply defined** because with more point sources, *destructive interference is more likely* – there are *more nodal lines between each antinodal line*
- The fringes are **brighter** because with more point sources, *more light waves* are able to contribute towards constructive interference, causing *increased amplitude*

Since the fringes are *more widely spread*, **x is no longer small compared to L** so the assumption that $sin(\theta) \approx \frac{x}{L}$ no longer holds. To calculate path difference, we must therefore use the following equation instead:

$$\boldsymbol{Path\ difference = dsin(\theta)}$$

[10] Image sourced from www.physics.uq.edu.au/people/mcintyre/php/laboratories/index.php?e=14

Again, by incorporating the path difference equations for *nodes* and *antinodes* we can obtain the following for any value $\mathbf{0} \leq \boldsymbol{\theta} < 90°$:

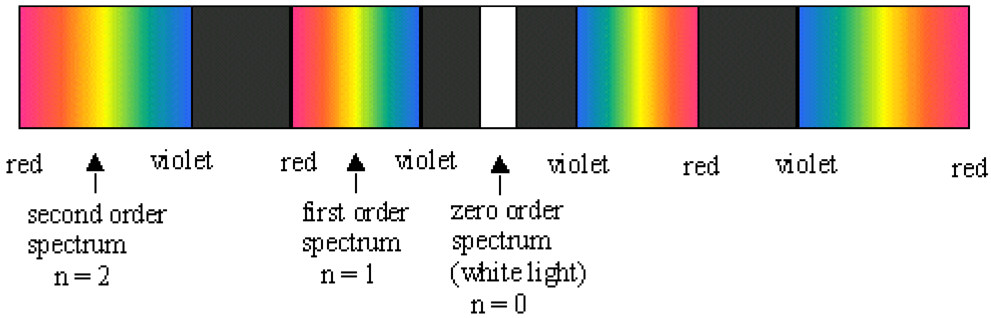
$$\boldsymbol{For\ any\ antinodal\ line\ (bright\ fringe) \rightarrow\ n\lambda = dsin(\theta)}$$

$$\boldsymbol{For\ any\ nodal\ line\ (dark\ fringe)\ \rightarrow\ (n - \frac{1}{2})\lambda = dsin(\theta)}$$

In general, when it comes to interference patterns, it is always better to use these two equations – and not the previous ones – in your calculations to avoid making incorrect approximations

White light contains all colours of light from the *visible light* part of the electromagnetic spectrum. Each different colour of light has a *different wavelength*. This means that when white light is shone through a *diffraction grating*, each bright fringe will consist of a **spectrum** of visible light, ranging from *red to violet*. Because red light has the *largest wavelength*, it will diffract to the greatest extent and therefore, within each bright fringe, it will be found **furthest from the central antinodal line**.

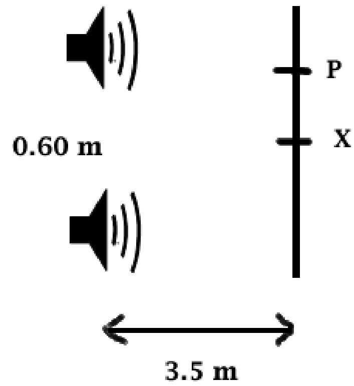
The only bright fringe that **does not show a spectrum** of visible light is the *central antinodal line*. This is because the light waves interfering to produce the central antinodal line *do not diffract* (they travel in a straight line) so *all the colours in white light* combine to give a **white central fringe** as shown below:



[11]

[11] Image sourced from www.animatedscience.co.uk/ks5_physics/general/Oscillations%20&%20Waves,%20Reflection%20&%20Refraction/Waves%20%28Part%202%29.htm

Section 2 - Questions



Sound waves travel at 330 ms^{-1}. Light waves travel at $3.0 \times 10^8\ ms^{-1}$.

1. Two speakers are placed 0.60 m apart and both emit the same loudness of sound at 510 Hz as shown in the diagram to the right.
   i) What is the wavelength of the sound waves produced? Give your answer to the correct number of significant figures.
   $v = f\lambda \rightarrow \lambda = \frac{v}{f} = 330ms^{-1} \div 510\ Hz = 0.65\ m\ (2sf)$
   ii) A loud sound is heard when positioned exactly between the speakers at point X but this sound fades out as you move either up or down the line. Explain why.
   The two point sources produce sound of identical frequency and amplitude which creates an interference pattern of antinodes (points of constructive interference, in phase, whole number of wavelengths apart, maximum amplitude and loudness) and nodes (points of destructive interference, in phase, odd number of half wavelengths apart, minimum amplitude and no loudness). X is the central antinodal line -> loud, sound fades out as you move towards the adjacent nodal line.
   iii) Moving up the line from X, point P is the first point at which no sound is heard. Calculate the distance from X to P and explain any assumptions that are necessary in this calculation.
   P is the first nodal line so n = 1. Assuming that x << L (i.e. the angle θ is small), then
   $\left(n - \frac{1}{2}\right)\lambda = \frac{dx}{L} \rightarrow x = \frac{\left(n-\frac{1}{2}\right)\lambda L}{d} = \frac{\frac{1}{2}\times 0.65\ m \times 3.5\ m}{0.60\ m} = 1.9\ m\ (2sf)$ <- fairly bad assumption

2. Two laser lights are placed 0.5 cm apart and emit light of wavelength 586 nm. A pattern of bright and dark fringes does not appear on the screen a certain distance away. The interference pattern, however, does appear when one laser light of wavelength 586 nm is shone through two slits 0.5 cm apart in a piece of paper.
   i) Explain why the interference pattern does not appear on the screen with the two laser lights but does when a single laser light is shone through two slits.
   Two separate light sources are not coherent and therefore cannot interfere to produce a pattern. Monochromatic light shone through two slits is coherent and can interfere.
   ii) Calculate the path difference for the second nodal line when the laser is shone through the two slits.
   For n=2, $path\ difference = \left(n - \frac{1}{2}\right)\lambda = 1.5 \times 586\ nm = 879\ nm$
   iii) Given that the distance between any bright fringe and its adjacent dark fringe is 1.3 mm, find the distance of the screen from the point sources of light.
   This means that for the first nodal line, n = 1 and x = 0.013 m. Therefore $\left(n - \frac{1}{2}\right)\lambda = \frac{dx}{L} \rightarrow L = \frac{dx}{\left(n-\frac{1}{2}\right)\lambda} = \frac{0.005m \times 0.0013\ m}{\frac{1}{2}\times 586 \times 10^{-9} m} = 22.2\ m\ (3sf)$

3. Another beam of monochromatic light is shone through two slits which are 0.3 mm apart on a piece of paper. A screen is positioned 1.8 m away and shows up a pattern of bright and dark fringes. The distance between 7 of the bright fringes is 15 mm.

   i) Find the wavelength of this light wave.

   Distance between two adjacent bright fringes on the screen is x = 2.5 mm = 0.0025 m. Therefore distance between central antinode and antinodal line where n = 1 is 0.0025 m. $n\lambda = \frac{dx}{L} \rightarrow \lambda = \frac{dx}{nL} = \frac{0.0003\ m \times 0.0025\ m}{1 \times 1.8\ m} = 4.17 \times 10^{-7}\ m = 417\ nm\ (3sf)$

   ii) Give two ways in which the spacing of the fringes could be decreased.

   Either of: increase the slit spacing (d), decrease the distance from the screen (L) or decrease the wavelength (λ)

4. A diffraction grating has 7,000 slits per cm.

   i) Calculate the slit spacing of the diffraction grating.

$$d = \frac{1}{N} = \frac{1}{7000\ cm^{-1}} = 1.43 \times 10^{-4}\ cm = 1.43 \times 10^{-6}\ m$$

   ii) Find the wavelength of light which produces a first order bright fringe at 27°.

$$n\lambda = dsin(\theta) \rightarrow \lambda = \frac{dsin(\theta)}{n} = \frac{1.43 \times 10^{-6}\ m \times \sin(27°)}{1} = 649\ nm\ (3\ sf)$$

   iii) For this same wavelength, at what angle will the second order dark fringe be diffracted?

$$\left(n - \frac{1}{2}\right)\lambda = dsin(\theta) \rightarrow \sin(\theta) = \frac{\left(n - \frac{1}{2}\right)\lambda}{d} = \frac{(1.5 \times 649 \times 10^{-9}\ m)}{1.43 \times 10^{-6}\ m} = 0.681\ (3sf)$$

$$\rightarrow \theta = 42.9°\ (3sf)$$

5. A light of wavelength 670 nm is shone through a diffraction grating. The first bright fringe is diffracted at an angle of 26.0°.

   i) Calculate the number of slits per mm on the diffraction grating. Give your answer to the correct number of significant figures.

$$n\lambda = dsin(\theta) \rightarrow d = \frac{n\lambda}{\sin(\theta)} = \frac{1 \times 670 \times 10^{-9}\ m}{\sin(26°)} = 1.53 \times 10^{-6}\ m\ (3sf)$$

$$\rightarrow N = \frac{1}{d} = \frac{1}{1.53 \times 10^{-6}\ m} = 654{,}285\ per\ m\ (6sf) \div 1000\ = 654\ per\ mm\ (3sf)$$

   ii) Calculate the maximum number of complete bright fringes that could be produced from this grating.

   Maximum diffraction angle is 90°. Therefore last bright fringe when$n\lambda = dsin(90°) \rightarrow$ $n = \frac{d}{\lambda} = \frac{1.53 \times 10^{-6}\ m}{670 \times 10^{-9}\ m} =$ 2.28. Therefore there are 2 bright fringes either side of central bright fringe -> 5 in total.

   iii) Name one feature of this multi-slit interference pattern that would be different when compared to a double-slit interference pattern and explain your answer.

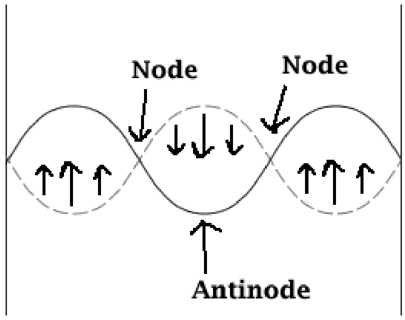
   Any three of: wider diffraction angle (smaller slit spacing), brighter light (more wave sources increase amplitude generated by constructive interference) or more defined bright fringes (more sources are less likely to constructively interfere).

Section 3 – Standing Waves and Music

When two *identical* waves move through each other in **opposite directions** (for example, when a wave reflects back off a barrier) they *interfere* and produce a *stationary wave* known as a **standing wave**. This wave has effectively *'trapped'* the energy of the two waves, oscillating between *kinetic* and *potential energy* forms.

As a result of wave interference, **standing waves** consist of:

- **Nodes**, or nodal points, which are parts of the standing wave that remain *stationary* at all times.
- **Antinodes**, or antinodal points, which are parts of the standing wave that have the *greatest displacement*. At the greatest displacement, the particles have *maximum potential energy*.



[12]

As you can see in Diagram 12, *any two wave particles between adjacent nodes* (in the same half wave) are **in phase**.

Musical instruments often produce their sound by generating *standing waves* in **strings** or **pipes**. Because *nodes* can only be generated at **fixed ends of string** or **closed ends of pipes** (no displacement) **and *antinodes* can only be generated at free ends of string or open ends of pipes** (maximum displacement), this limits the wavelengths that will *'fit'* into the string/pipe.

The frequency of a wavelength that can *'fit'* and hence establish a standing wave is known as a **resonant frequency**. The **fundamental frequency** (or 1st harmonic) is the resonant frequency which produces a standing wave of the *longest wavelength*. Many other standing waves can also be produced, and these are known as **harmonics** or **overtones**, as shown in the diagram on the following page.

Most importantly, **the wavelength of any harmonic is the fundamental wavelength divided by the number of the harmonic**. Consequently, there are *only odd numbered* harmonics for a closed pipe – these are the only standing wave patterns that will 'fit' inside to have a *node at one end* and an *antinode at the other*.

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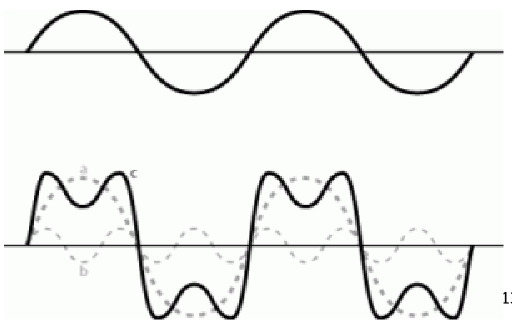
[12] Image sourced and edited from www.revisesmart.co.uk/physics/waves/standing-waves.html

Name	String	Closed pipe	Open pipe
1st Harmonic/ Fundamental Frequency	L; λ=2L	L; λ=4L	L; λ=2L
2nd Harmonic/ 1st Overtone	λ=L		λ=L
3rd Harmonic/ 2nd Overtone	λ=2L/3	λ=4L/3	λ=2L/3
4th Harmonic/ 3rd Overtone	λ=2L/4		λ=2L/4
5th Harmonic/ 4th Overtone	λ=2L/5	λ=4L/5	λ=2L/5

To produce a *single note*, musical instruments often generate a *combination* of these harmonics with *various amplitudes* (the fundamental frequency always having the highest amplitude). This combination gives the instrument a *unique tone*, known as **timbre**.


[13]

The **overall waveform** for a note produced by an instrument in this way has a *wavelength equal to the wavelength of the fundamental frequency*. The fundamental frequency – and hence **the pitch of the note** – can be increased by:

- **Shortening the length (L)** of the string or pipe
- **Increasing the tension** of the string -> this increases the *speed* of the standing wave and therefore increases its *frequency* as wavelength remains constant
- **Decreasing the heaviness (mass per metre)** of the string -> this also increases the *speed* of the standing wave and therefore increases its *frequency* as wavelength remains constant

A *doubling* of frequency results in a note produced that is one **octave** higher.

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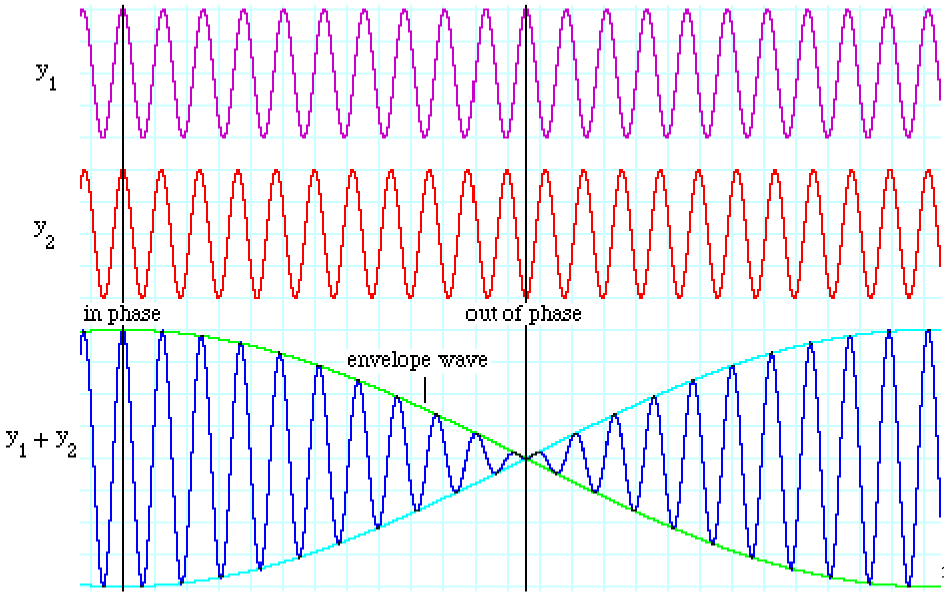
[13] Image sourced from http://curtismacdonald.com/harmony-and-timbre

Musical instruments can also generate **beats**. This is when two sources produce *sound waves* of **slightly different frequency** so the two waves interfere and *drift in and out of phase*. This creates a resultant wave pattern with a frequency equal to the *difference in frequency of the two sources* (f_1 and f_2):

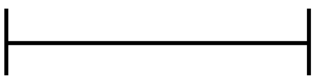
$$f_{Beat} = |f_1 - f_2|$$

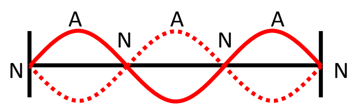


[14]

Section 3 - Questions

1. A guitar string has a length of 0.74 m and is plucked, generating a wave of speed 500 ms^{-1}.
   i) Explain how this can set up a standing wave.
   The wave generated can reflect back off the fixed ends of string, causing two identical waves moving in opposite directions to interfere and generate and standing wave pattern of nodes and antinodes.
   ii) Complete the below showing the third harmonic (second overtone) of the string. Label the nodes and antinodes.





[14] Image sourced from http://www.animations.physics.unsw.edu.au/jw/beats.htm

iii) Calculate the frequency of the third harmonic. Give your answer to the correct number of significant figures.

$$f = \frac{v}{\lambda} = \frac{v}{\frac{2L}{3}} = \frac{500\ ms^{-1}}{\frac{2}{3} \times 0.74\ m} = \ 1000\ Hz\ (2sf)$$

iv) Calculate the length of string that would generate a fundamental frequency of 540 Hz.

$$\lambda = \frac{v}{f} \to 2L = \frac{v}{f} = \frac{500\ ms^{-1}}{540\ Hz} = 0.926\ m\ (3sf) \to L = 0.463\ m\ (3sf)$$

2. Blowing into an open pipe generates a tone at a fundamental frequency of 800 Hz. The speed of sound in the pipe is 330 ms^{-1}.

i) Complete the diagram below showing the fundamental frequency (first harmonic) in this pipe.

ii) Calculate the length of the pipe.

$$\lambda = \frac{v}{f} = \frac{330\ ms^{-1}}{800\ Hz} = 0.4125\ m\ \to 2L = \lambda = 0.4125\ m\ \to L = 0.206\ m\ (3sf)$$

iii) The bottom of the pipe is then closed off. Explain how this will affect the fundamental frequency and calculate the new fundamental frequency for this closed pipe.

When the bottom of the pipe is closed off, the fundamental frequency decreases. This is because with a closed end, the wavelengths that can 'fit' into the pipe to create a standing wave change. There must now be a node at the closed end (antinode at the open end). Thus, instead of half a wavelength fitting in to generate the fundamental frequency in the open pipe, the closed pipe can fit a quarter of a wavelength to generate the fundamental frequency. Therefore,

$$f = \frac{v}{\lambda} = \frac{v}{4L} = \frac{330\ ms^{-1}}{4 \times 0.206\ m} = 400\ Hz$$

3. Two violin strings, string A and string B, are plucked simultaneously and generate standing waves of speed 575 ms^{-1}. The fundamental frequency produced in string B is 420 Hz, which produces 'beats' of 4.00 Hz with the fundamental frequency produced in string A.

i) Explain how these 'beats' are formed.

The two strings produced notes of slightly different frequencies, causing the sound waves produced to interfere and drift in and out of phase. This creates a 'beat' pattern with frequency equal to the difference in frequency of the two notes.

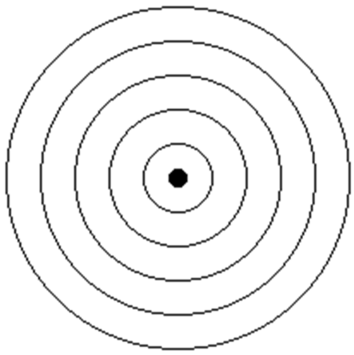
ii) Given that string A is shorter than string B, find the length of string A. Give your answer to the correct number of significant figures.

String A is shorter therefore its fundamental wavelength will be shorter, giving it a higher frequency. Thus, $f_A = 420\ Hz + 4\ Hz = 424\ Hz\ \to\ \lambda = 2L = \frac{v}{f} = \frac{575\ ms^{-1}}{424\ Hz} = 1.36\ m\ (3sf) \to L = 0.678\ m\ (3sf)$

Section 4 – The Doppler Effect

When a source that produces waves is **moving**, it causes an *apparent shift in frequency* called the **Doppler Effect**. If the object is moving *towards* an observer, the frequency appears *higher* than it is and if the object is moving *away* from an observer, the frequency appears *lower*.

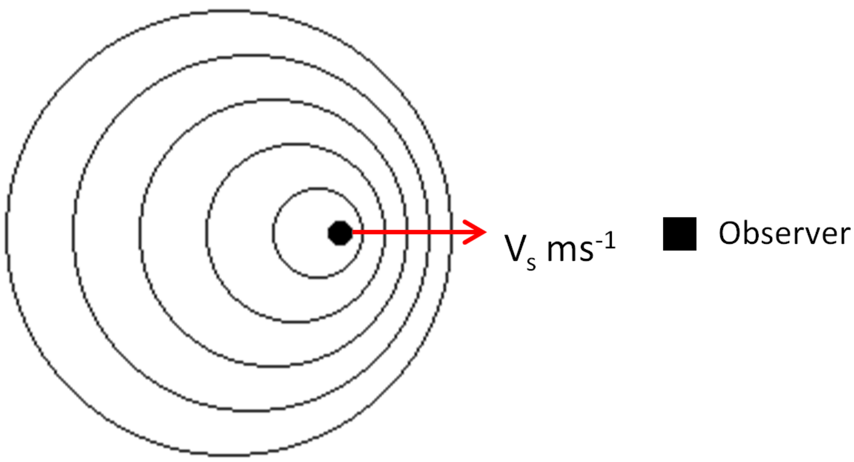
To explain this phenomenon, consider a *stationary point source* emitting waves of frequency $\boldsymbol{f}\ Hz$ at a speed of $\boldsymbol{v_w}\ ms^{-1}$. A **certain number of waves are emitted each second** over the **same distance** *in front of and behind* the source. This is shown in Diagram 15.


[15]

Now, as shown in Diagram 16, if the point source starts **moving** at $\boldsymbol{v_s}\ ms^{-1}$, you can see that **directly in front of the source**:

- The **relative movement** of the source *towards a stationary observer* means that, *directly in front of the source*, the **same number of waves** is emitted per second but over a **shorter distance**. This causes the waves to *'bunch up'* and hence results in an *apparent decrease in wavelength* and consequent **increase in frequency**. This increased apparent frequency (**f'**) *directly in front of the source* is given by:

$$\boldsymbol{f' = \frac{v_w f}{v_w - v_s}}$$



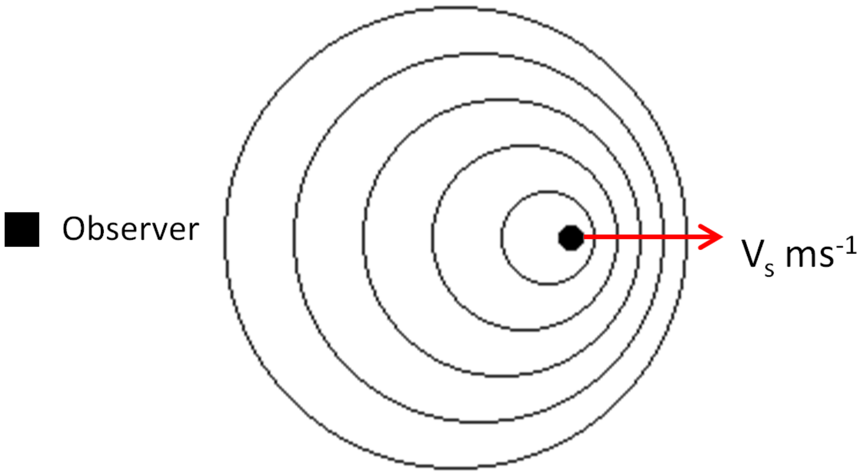
[16]

[15] Image sourced from http://richardtaylor.co.uk/doppler/index.html

In contrast, as shown in Diagram 17, you can see that **directly behind the source**:

- The **relative movement** of the source *away from a stationary observer* means that, *directly behind the source*, the **same number of waves** is emitted per second but over an **increased distance.** This causes the waves to *'spread out'* and hence results in an *apparent increase in wavelength* and consequent **decrease in frequency**. This decreased apparent frequency (**f'**) *directly behind the source* is given by:

$$f' = \frac{v_w f}{v_w + v_s}$$



[17]

As you can see from either equation, **the apparent change in frequency will only be evident if v_s is significant when compared to v_w.**

The Doppler Effect has several applications:

- **Speed detectors** reflect waves off cars and measure the change in frequency as a means for measuring the *speed of the car*
- Visible light emitted from distant stars which are *moving away* from Earth has an *increased wavelength* and thus 'shifts' towards the red end of the spectrum – this is known as **red shift**
- **Sonic booms** are created if a source of sound waves (e.g. a jet plane) moves *faster than the speed of sound*, causing the waves to '*bunch up'* behind the source and form *a shock wave*.

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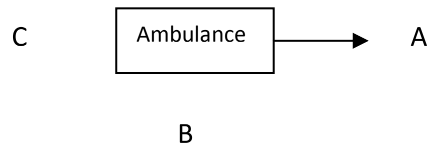
[16] Image sourced and edited from http://richardtaylor.co.uk/doppler/index.html
[17] Image sourced and edited from http://richardtaylor.co.uk/doppler/index.html

Section 4 – Questions

The speed of sound in air is 330 ms^{-1}. The speed of light is $3 \times 10^8\ ms^{-1}$



1. An ambulance is moving at 24 ms^{-1} as shown in the diagram to the right. Its siren produces sound waves of 47 Hz.
   - i) Explain why someone at B would hear the siren at 47 Hz but someone at A (who the ambulance is approaching) would hear the siren at a higher pitch.

     At B, there is no relative movement between the ambulance and the observer so the wavelength (and hence frequency) remains normal. At A, since the ambulance is moving towards the observer, the same number of sound waves is emitted over a smaller distance. This effectively causes the waves to 'bunch' up and have a smaller wavelength (and hence higher frequency at the same speed) – the Doppler Effect.
   - ii) Calculate the frequency of sound heard by someone at A.

     $$f' = \frac{v_w f}{v_w - v_s} = \frac{47\ Hz \times 330\ ms^{-1}}{330\ ms^{-1} - 24\ ms^{-1}} = 50.7\ Hz\ (3sf)$$
   - iii) Humans generally cannot hear sound at 20 Hz or below, nor at 20 kHz or above. Calculate the theoretical minimum speed the ambulance would have to move so that someone at C would not be able to hear the siren.

     $$f' = \frac{v_w f}{v_w + v_s} \to 20\ Hz = \frac{330\ ms^{-1} \times 47\ Hz}{330\ ms^{-1} + v_s} \to v_s + 330\ ms^{-1} = \frac{330\ ms^{-1} \times 47\ Hz}{20\ Hz} \to$$
     $$v_s = 445.5\ ms^{-1}$$

2. A theoretical distant star emits visible light of wavelength 618 nm. The same light from the star – when seen from Earth – appears more red, with a wavelength of 640 nm.
   - i) Calculate the frequency of light which the star emits. Give your answer to the correct number of significant figures.

     $$f = \frac{v}{\lambda} = \frac{3 \times 10^8\ ms^{-1}}{618 \times 10^{-9}\ m} = 4.85 \times 10^{14}\ Hz\ (3sf)$$
   - ii) Explain why the visible light is 'red shifted' when it reaches Earth

     The star is moving away from Earth and therefore, by the Doppler Effect, there is the same number of waves emitted per second over a longer distance, causing an apparent increase in wavelength -> shift towards the red end of the visible light spectrum.
   - iii) Calculate how fast the star is moving.

     $$f' = \frac{v_w f}{v_w + v_s} \to \frac{3 \times 10^8\ ms^{-1}}{640 \times 10^{-9}\ m} = \frac{3 \times 10^8\ ms^{-1} \times 4.85 \times 10^{14}\ Hz}{3 \times 10^8\ ms^{-1} + v_s}$$
     $$\to v_s + 3 \times 10^8\ ms^{-1} = 640 \times 10^{-9}\ m \times 4.85 \times 10^{14}\ Hz$$
     $$\to v_s = 1.07 \times 10^7\ ms^{-1}$$
   - iv) Under what circumstances may the light be 'violet shifted'?

     If the star is moving towards the Earth, by the Doppler Effect, there will the same number of waves emitted per second over a smaller distance – they will 'bunch' up and cause an apparent decrease in wavelength, shifting the light towards the violet end of the spectrum.