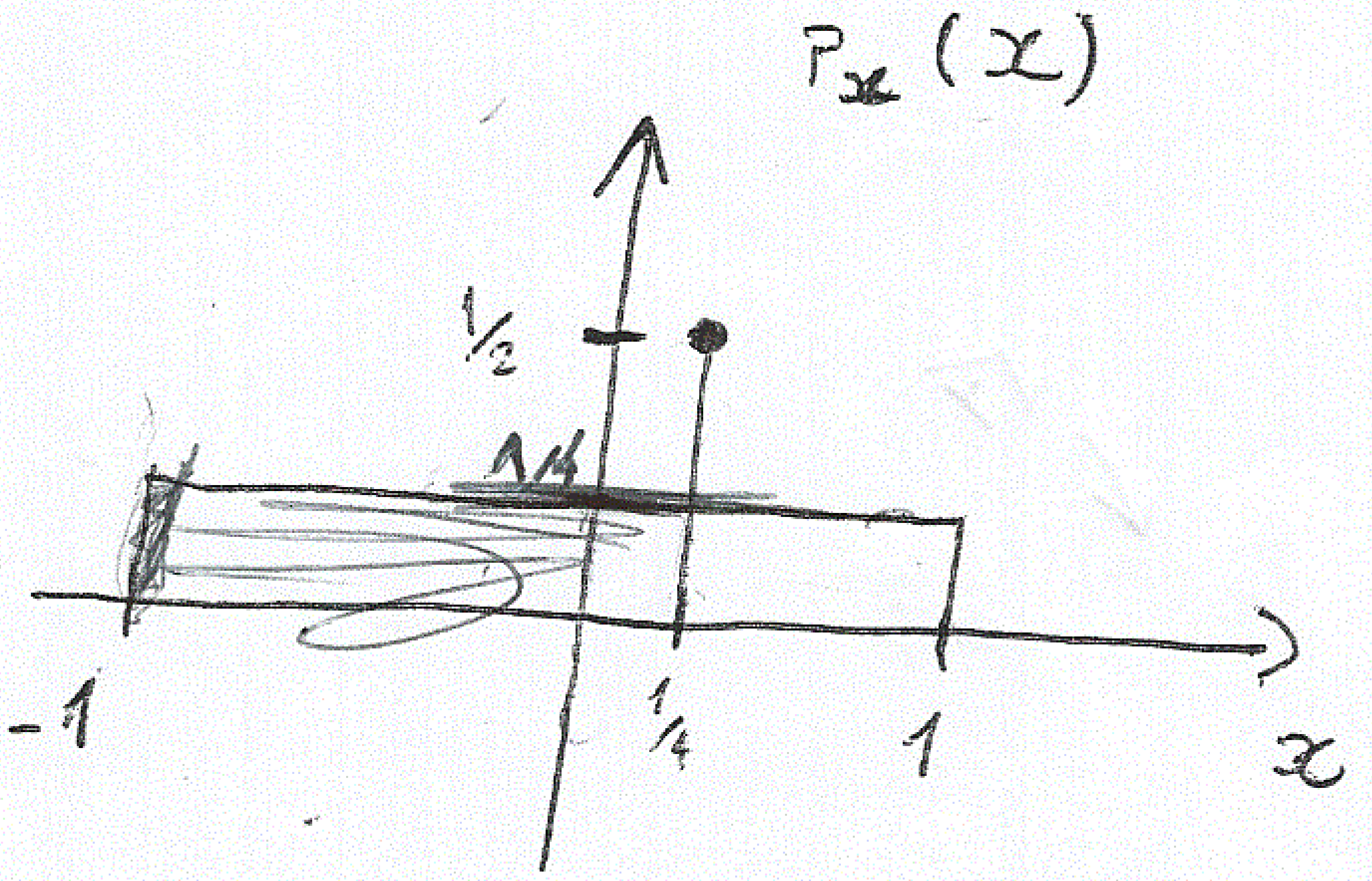
Problema 1

$$P_x(x) = \left[\frac{1}{4} + \frac{1}{2}\,\delta\left(x - \frac{1}{4}\right)\right] \cdot \left[u(x+1) - u(x-1)\right]$$

a) FDP de VA (x):



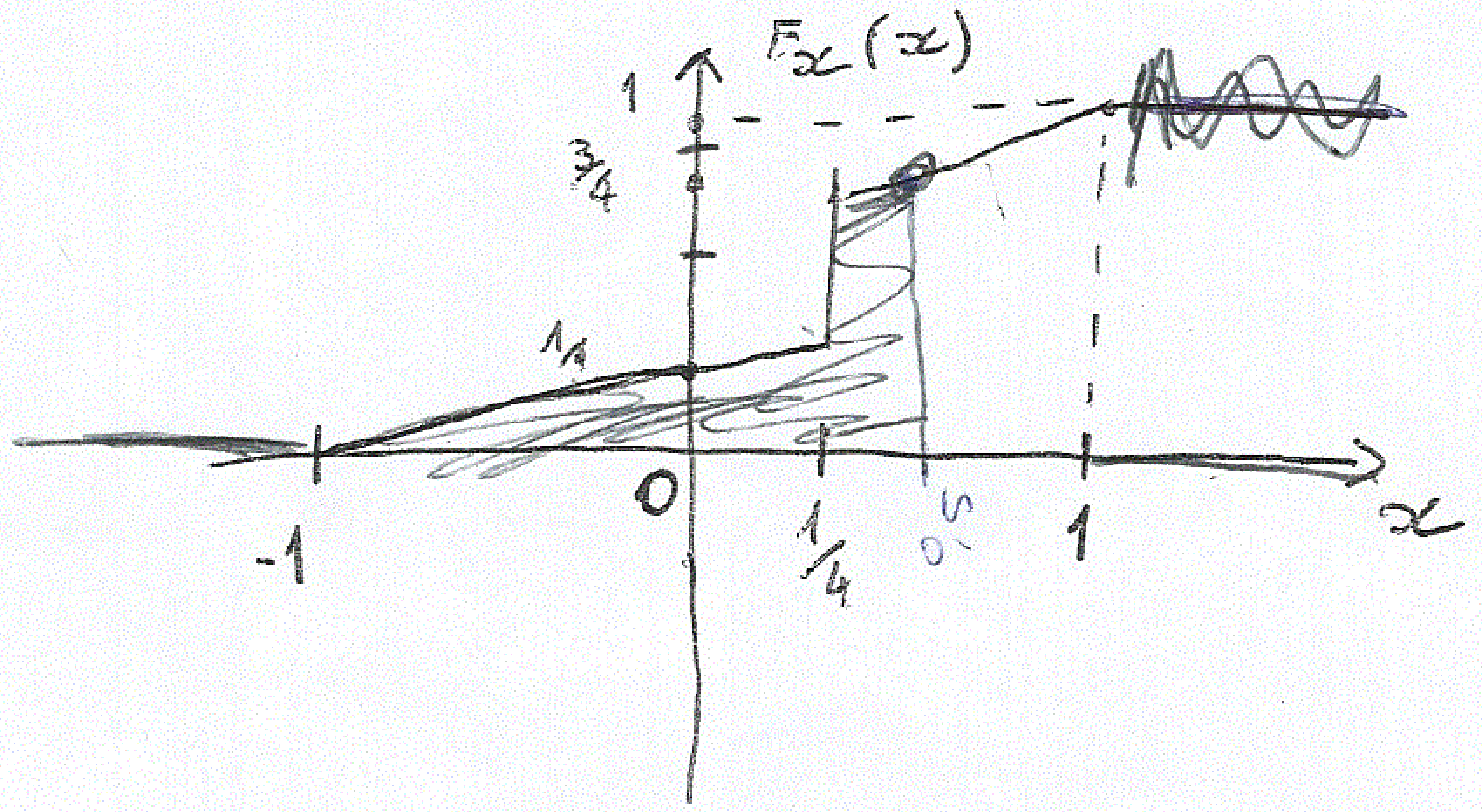
igual a $\frac{1}{2}$ em $x = \frac{1}{4}$

$$F_x(x) = \int_{-\infty}^{x} P_x(\lambda)\, d\lambda = \int_{-1}^{x} \left[\frac{1}{4} + \frac{1}{2}\,\delta\left(\lambda - \frac{1}{4}\right)\right] d\lambda =$$

$$\frac{1}{4}\left[\lambda\right]_{-1}^{x} = \frac{(x+1)}{4}$$

$$F_x(x) = \begin{cases} \dfrac{(x+1)}{4}, & x \in \left[-1, \frac{1}{4}\right[\\ \dfrac{(x+1)}{4} + \dfrac{1}{2}, & x \in \left[\frac{1}{4}, 1\right] \end{cases}$$

FDPC de VA (x)

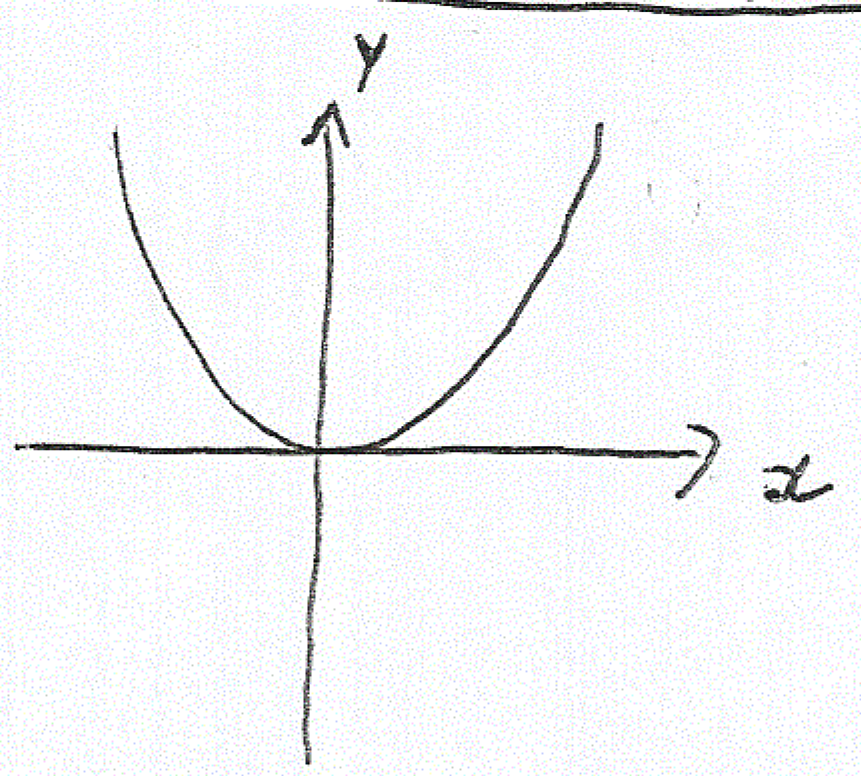


b) $P_x(x \geq 0{,}5) = 1 - P(x < 0{,}5) = 1 - \left[\frac{(0{,}5+1)}{4} + \frac{1}{2}\right] = 0{,}125 = 12{,}5\%$

(1)

Problema 1

c) $\boxed{Y = X^2 \Leftrightarrow X = \sqrt{Y}}$



$$P_Y(y) = P_X(x) \cdot \left|\frac{\partial \sqrt{y}}{\partial y}\right| = P_X(\overset{\sqrt{y}}{x}) \cdot \frac{1}{2\sqrt{y}} = \left[\frac{1}{4} + \frac{1}{2}\,\delta\left(\sqrt{y} - \frac{1}{4}\right)\right] \cdot \left[u(\sqrt{y}+1) - u(\sqrt{y} - 1)\right] \cdot \frac{1}{2\sqrt{y}}$$

d) $$F_Y(y) = \int_{-\infty}^{y} P_Y(\lambda)\, d\lambda = \int_0^y \left(\left[\frac{1}{4} + \frac{1}{2}\,\delta\left(\sqrt{\lambda} - \frac{1}{4}\right)\right] \cdot \left[u(\sqrt{\lambda}+1) - u(\sqrt{\lambda} - 1)\right] \cdot \frac{1}{2\sqrt{\lambda}}\, d\lambda = \int_0^y \left(\frac{1}{4} \cdot \frac{1}{2\sqrt{\lambda}}\right) d\lambda + \frac{1}{2}\,\delta\left(\sqrt{\lambda} - \frac{1}{4}\right)$$

$$= \int_0^y \left(\frac{1}{8\sqrt{\lambda}}\right) d\lambda = \frac{1}{8}\left(\int_0^y \left(\frac{1}{\sqrt{\lambda}}\right)\right) d\lambda = \frac{1}{8}\left[2\sqrt{\lambda}\right]_0^y = \frac{1}{8}\left[2\sqrt{y} - 2\sqrt{0}\right]$$

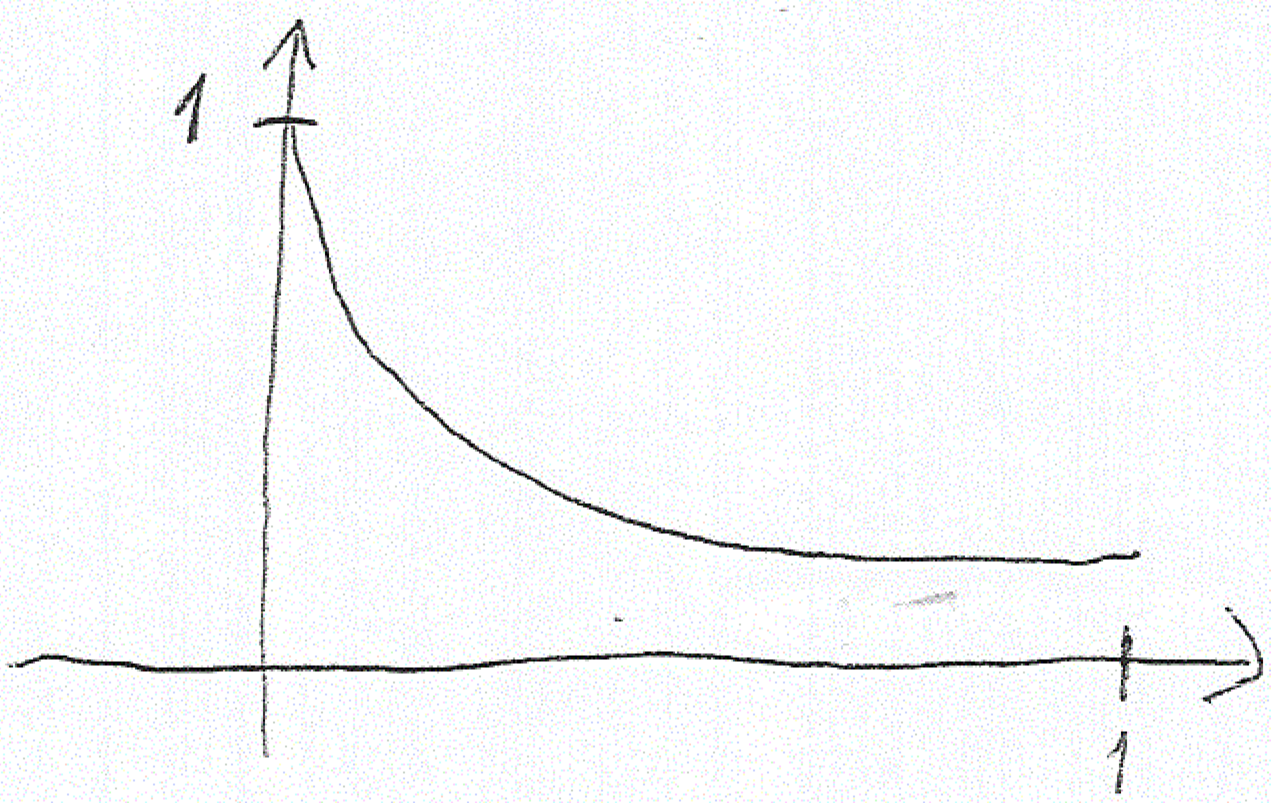
$$= \frac{2\sqrt{y}}{8} = \frac{1\sqrt{y}}{4}$$

$$F_Y(y) = \begin{cases} \dfrac{\sqrt{y}}{4}, & y \in [0, \frac{1}{16}[\\ \dfrac{\sqrt{y}}{4} + \dfrac{1}{2}, & y \in [\frac{1}{16}, 1] \end{cases}$$

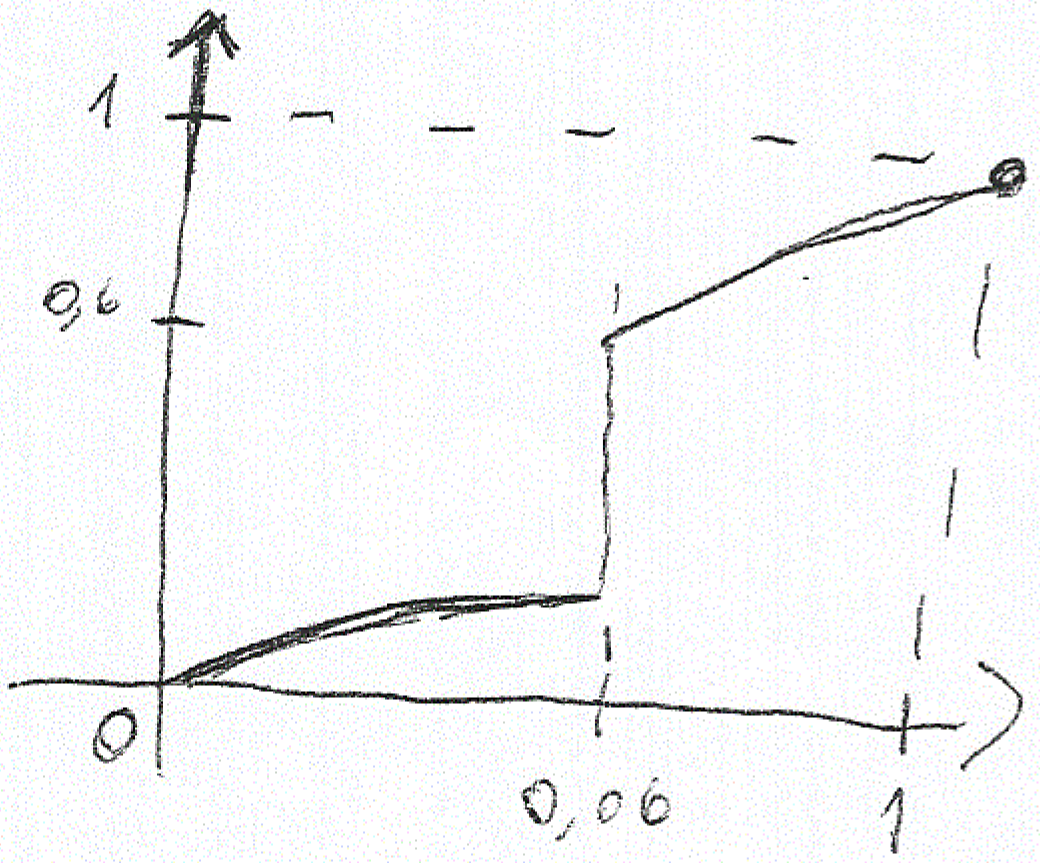
R: Quando $y = \frac{1}{16}$ o valor do delta dirac é $\frac{1}{2}$, de resto é $\varnothing$.

Problema 1

e) FDP VA (Y)



FDPC VA (Y)



Problema 2)

a) $\displaystyle\int_{-\infty}^{+\infty}\int_{x}^{+\infty} f_{xy}(x,y)\cdot dy\,dx = 1 \Leftrightarrow$

$x < y$

$\Leftrightarrow \displaystyle\int_{0}^{+\infty}\int_{x}^{+\infty} 6\times 10^{-6}\cdot e^{-0,001x}\cdot e^{-0,002y}\cdot dy\,dx =$

$= 6\times10^{-6}\displaystyle\int_{0}^{+\infty} e^{-0,001x}\left(\int_{x}^{+\infty} e^{-0,002y}\,dy\right)dx =$

$= 6\times10^{-6}\displaystyle\int_{0}^{+\infty} \frac{e^{-0,002x}}{0,002}\cdot e^{-0,001x}\cdot dx =$

$= 0,003\displaystyle\int_{0}^{+\infty} e^{-0,003x}\cdot dx = 0,003\left[\frac{0-1}{-0,003}\right] =$

$= \cancel{0,003}\cdot\dfrac{1}{\cancel{0,003}} = 1$ c.q.d

b) $P(X<1500, Y<2500) = \displaystyle\int_{0}^{1500}\int_{x}^{2500} f_{xy}(x,y)\cdot dy\,dx =$

$= 6\times10^{-3}\displaystyle\int_{0}^{1500}\left(\frac{e^{-0,002x}-e^{-5}}{0,002}\right)\cdot e^{-0,001x}\cdot dx =$

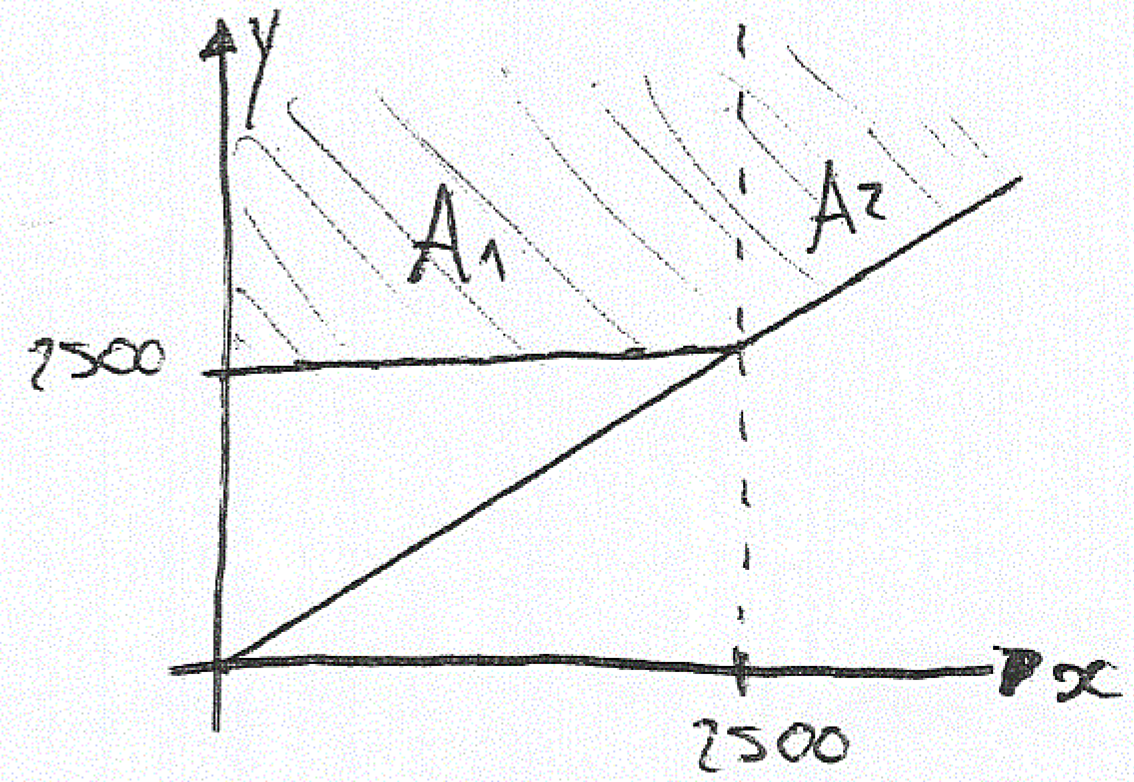
$= 0,003\displaystyle\int_{0}^{1500} e^{-0,003x} - e^{-5}\cdot e^{-0,001x}\cdot dx =$

$$= 0{,}003 \cdot \left(\int_0^{1500} e^{-0{,}003x} \cdot dx - e^{-5} \int_0^{1500} e^{-0{,}001x} \cdot dx \right) =$$

$$= 0{,}003 \left(\frac{1 - e^{-4{,}5}}{0{,}003} - e^{-5} \cdot \frac{1 - e^{-1{,}5}}{0{,}001} \right) =$$

$$= 0{,}973 = 97{,}3\%$$

c)



$A_1: x < 2500$

$A_2: x \geq 2500$

$$P(Y > 2500) = \int_0^{2500} \int_{2500}^{+\infty} f_{XY}(x,y) \cdot dy\,dx + \int_{2500}^{+\infty} \int_{x}^{+\infty} f_{XY}(x,y) \cdot dy\,dx =$$

$$= 6\times10^{-6} \left(\int_0^{2500} e^{-0{,}001x} \cdot dx \int_{2500}^{+\infty} e^{-0{,}002y}\, dy + \int_{2500}^{+\infty} e^{-0{,}001x} \cdot dx \int_{x}^{+\infty} e^{-0{,}002y} \cdot dy \right)$$

$$= 6\times10^{-6} \left(\int_0^{2500} \frac{e^{-5}}{0{,}002} \cdot e^{-0{,}001x} \cdot dx + \int_{2500}^{+\infty} \frac{e^{-0{,}002x}}{0{,}002} \cdot e^{-0{,}001x} \cdot dx \right) =$$

$$= 6\times10^{-6} \left(\frac{e^{-5}}{0{,}002} \times \frac{1 - e^{-2{,}5}}{0{,}001} + \frac{1}{0{,}002} \cdot \frac{e^{-7{,}5}}{0{,}003} \right) = 0{,}0191 =$$

$$= 1{,}91\%$$

$$f_Y(y) = \int_0^y 6\times10^{-6} \cdot e^{-0{,}001x} \cdot e^{-0{,}002y} \cdot dx =$$

$$= 6\times10^{-6} \cdot e^{-0{,}002y} \cdot \left[\frac{e^{-0{,}001x}}{-0{,}001}\right]_0^y =$$

$$= 6\times10^{-6} \cdot e^{-0{,}002y} \cdot \left(\frac{1 - e^{-0{,}001y}}{0{,}001}\right) =$$

$$= 6\times10^{-3} \cdot e^{-0{,}002y} \left(1 - e^{-0{,}001y}\right)$$

d) $P(Y > 2500 \mid X = 1000) = \int_{2500}^{+\infty} f_Y(y|x) \cdot dy = *$

$$f_Y(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

$$f_X(x) = \int_x^{+\infty} 6\times10^{-6} \cdot e^{-0{,}001x} \cdot e^{-0{,}002y} \cdot dy =$$

$$= 6\times10^{-6} \cdot e^{-0{,}001x} \cdot \left[\frac{e^{-0{,}002y}}{-0{,}002}\right]_x^{+\infty} =$$

$$= 6\times10^{-6} \cdot e^{-0{,}001x} \cdot \left[\frac{e^{-0{,}002x}}{0{,}002}\right] =$$

$$= 0{,}003 \cdot e^{-0{,}003x}$$

$$f_Y(y|x) = \frac{6\times10^{-6} \cdot e^{-0{,}001x} \cdot e^{-0{,}002y}}{0{,}003 \cdot e^{-0{,}003x}} = 0{,}002 \cdot e^{0{,}002x} \cdot e^{-0{,}002y}$$

$$* = \int_{2500}^{+\infty} 0{,}002 \cdot e^{0{,}002(1000)} \cdot e^{-0{,}002} \, dy =$$

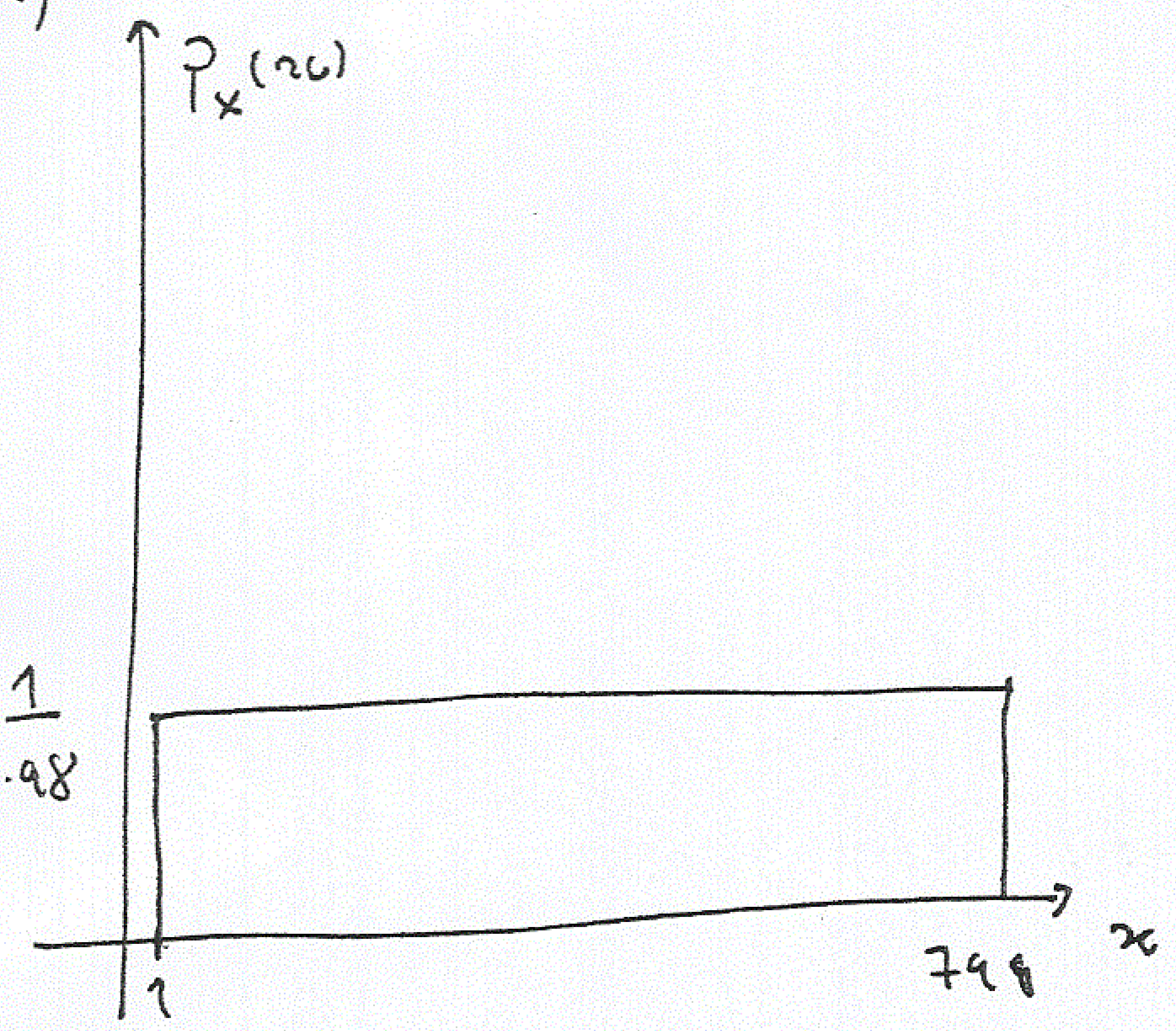
$$= 0{,}002 \cdot e^{2} \cdot \left[\frac{e^{-0{,}002y}}{-0{,}002} \right]_{2500}^{+\infty} = 0{,}002 \cdot e^{2} \cdot \frac{e^{-5}}{0{,}002} =$$

$$= 0{,}0498 = 4{,}98\%$$

Como $f_Y(y) \cdot f_X(x) \neq f_{XY}(x,y)$, estas son dependentes

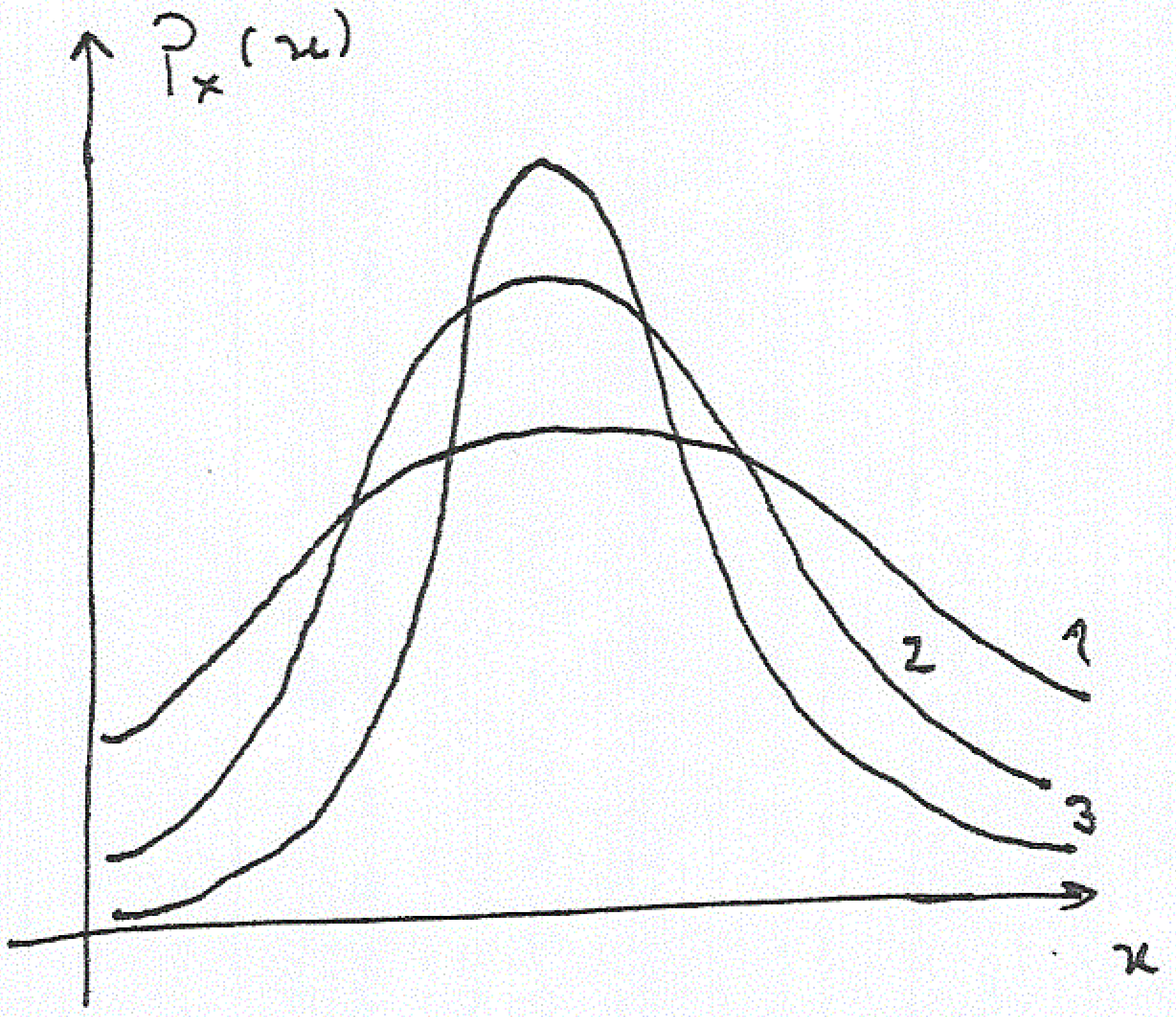
3

1)



$P_x(x) = \frac{1}{798}\left(u(x-1) - u(x-799)\right)$

igual a $P_y(y)$



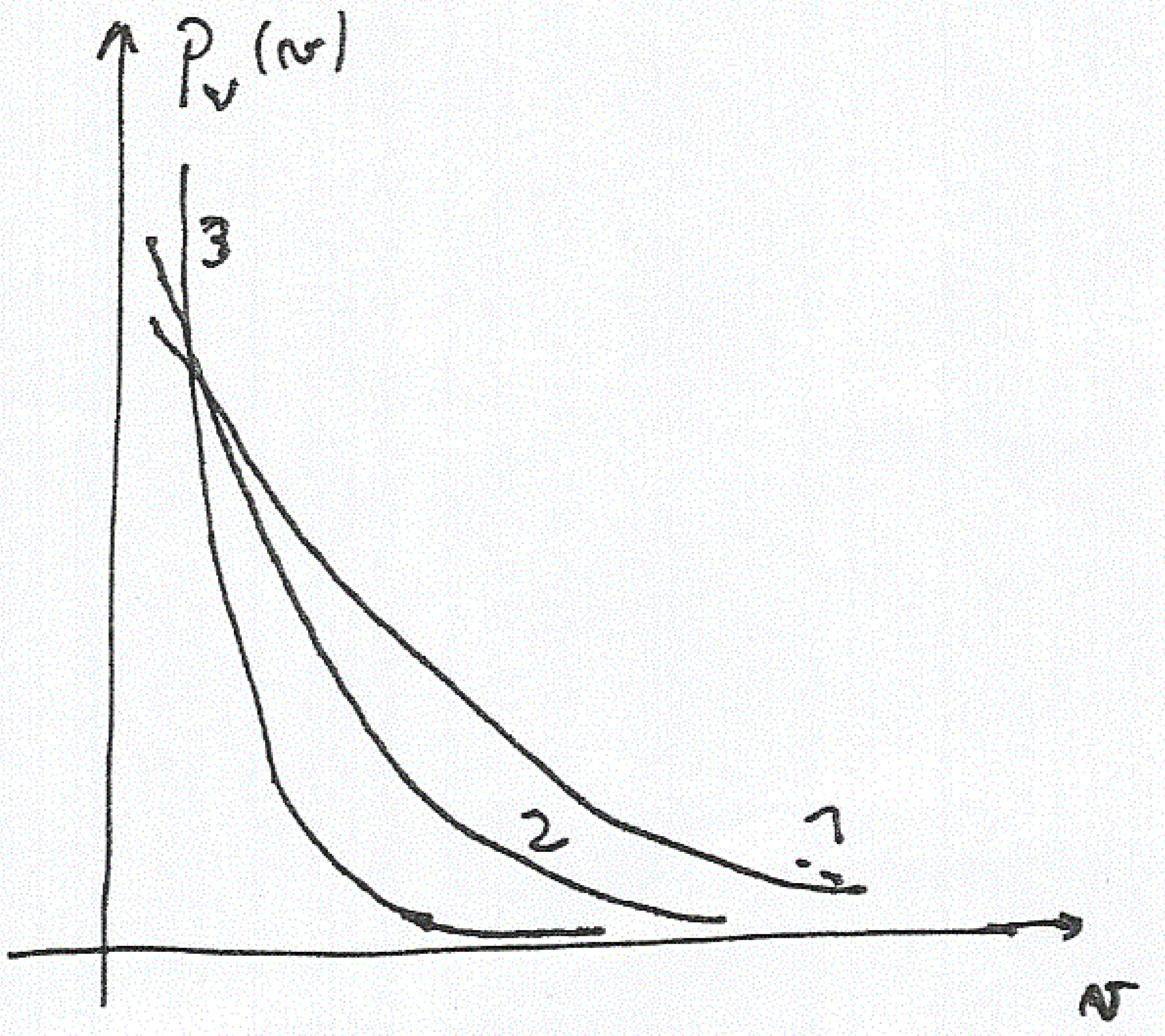
$P_x(x) = P_y(y) = \frac{1}{\sqrt{2\pi}\cdot\sigma_y} \cdot e^{\left(-\frac{(y-m_y)^2}{2\cdot\sigma_y^2}\right)}$

Niveis:

1 - $\sigma_y = 200$

2 - $\sigma_y = 100$

3 - $\sigma_y = 50$



$P_v(v) = m_v e^{-(m_v \cdot v)}$

Niveis

1 - $m_v = 50$

2 - $m_v = 100$

3 - $m_v = 200$

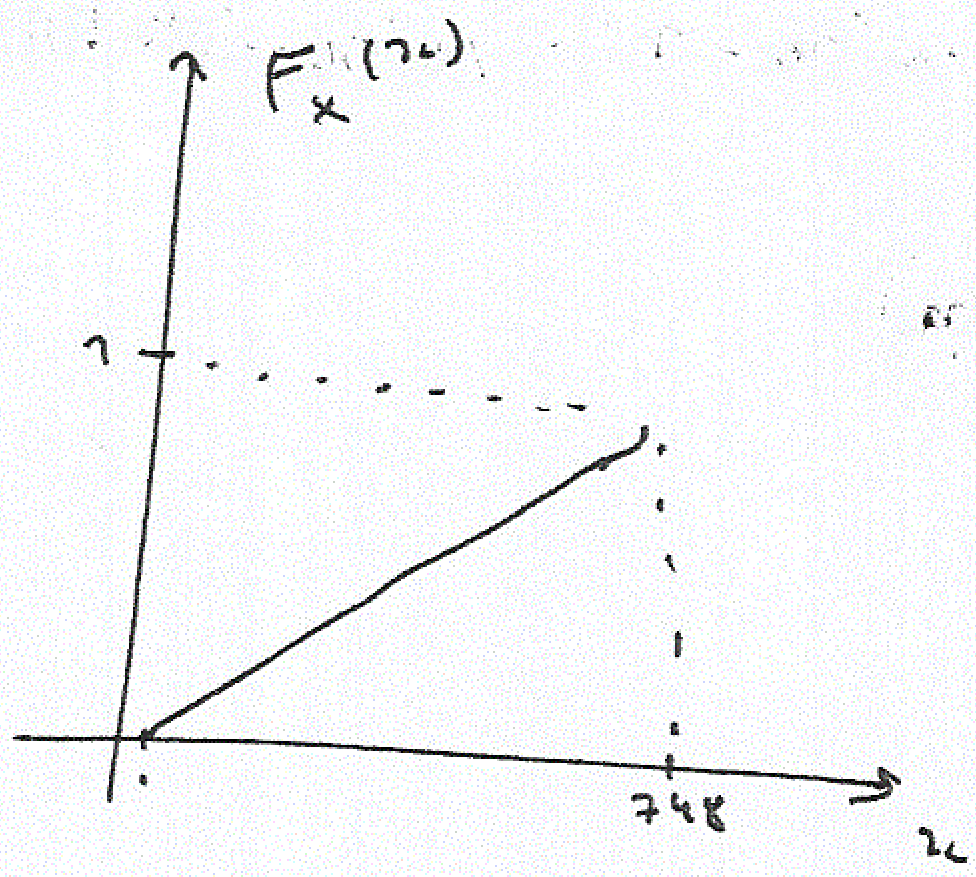
$x = 0$

$P_v(y) = 50$

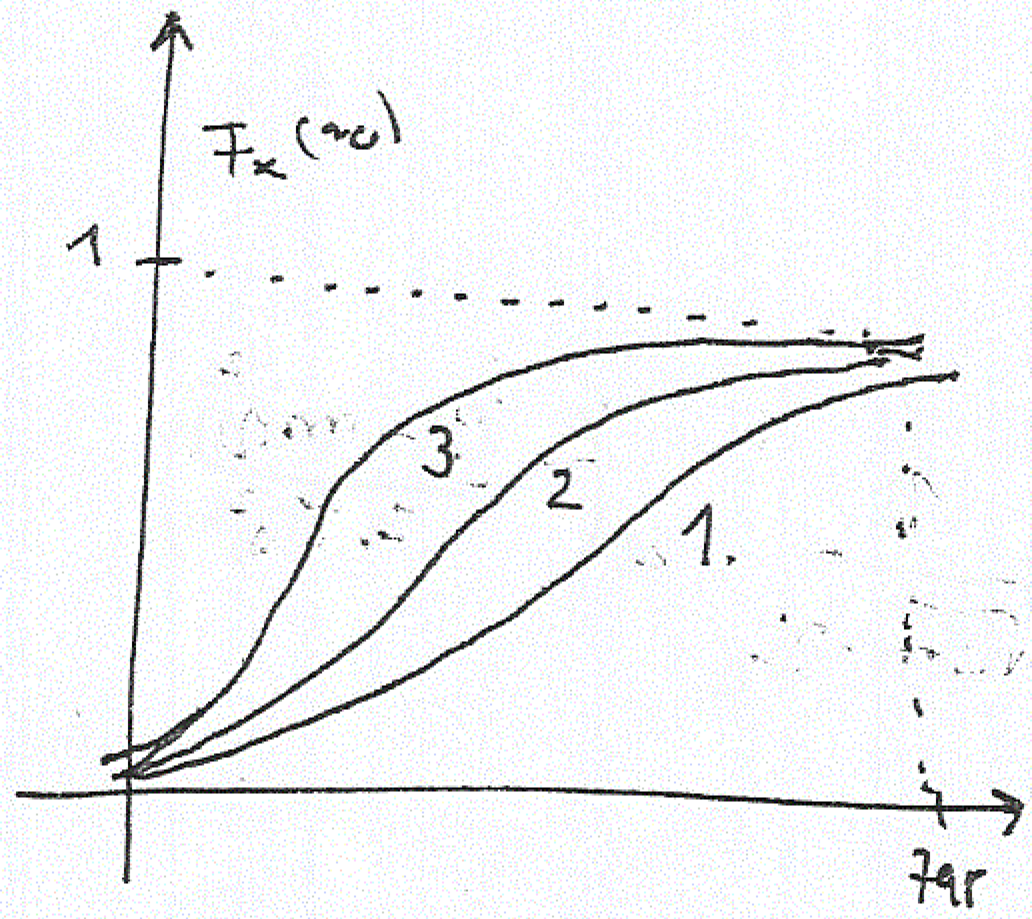
$= 100$

$= 200$

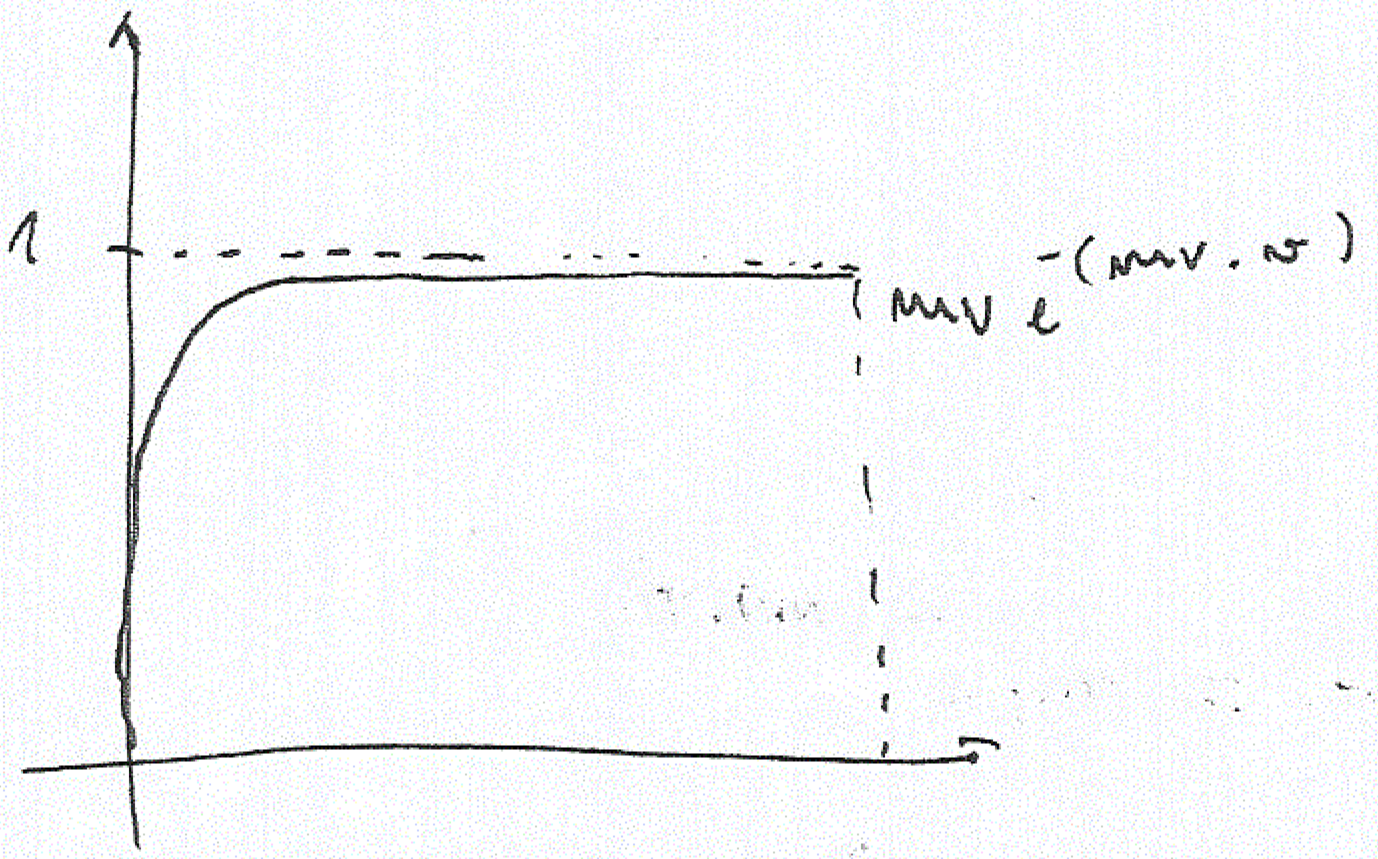
Representação FDPC's



Uniforme



GAUSSIANA



b) $\int_0^{180} P_V(v) \cdot dv = \int_0^{180} 200 \cdot e^{-(200 \cdot v)} \cdot dv =$

$= 200 \int_0^{180} e^{-200v} \cdot dv = \cancel{200} \cdot \left[\frac{e^{-200v}}{-\cancel{200}} \right]_0^{180} =$

$= 1 - e^{-200 \times 180} =$

$= -(e^{-200 \cdot 180} - 1) \approx 1$

c)

$\left[-e^{-200v} + 1 \right] = \left[1 - e^{-200v} \right]$

$\left[1 - e^{-200v} \right] = 0.2 \quad (20\%)$

$-e^{-100} = 0,2 - 1$

$\Leftrightarrow -e^{-100v} = 0,8$

$\Leftrightarrow -100v = -\ln(0,8)$

$\Leftrightarrow 100v = \frac{\ln(1,25)}{200} \approx 0,002 \text{ b/s}$

d) $-mv \cdot \ln(U)$

$P_x = U \times 798 + 1$ (uniforme)

centrar em mx

$P_x = P_y = (\sigma_x \cdot \sqrt{-2\ln(U_1)} \times \cos(2\pi \cdot U_2)) + mx$

e) $F_x(x) = \int_1^x \frac{1}{798} \, d\lambda = \frac{1}{798} \int_1^x 1 \, d\lambda = \frac{1}{798} [\lambda]_1^x = \frac{x-1}{798}$

$P(200 < X < 400) \rightarrow F_x(400) - F_x(200) = \left[\frac{400-1}{798} \right] - \left[\frac{200-1}{798} \right]$

(uniforme) $= \frac{399 - 199}{798} \simeq 0,28 = 25\%$

NIVEL 1 (GAUSSIANA)

$Q(5) - Q(1,5) = 5 \times 10^{-1} - 1,5 \times 10^{-1} = 12,25\%$

NIVEL 2 (GAUSSIANA)

$Q(5) - Q(3) = 5 \times 10^{-1} - 3 \times 10^{-5} = 25\%$

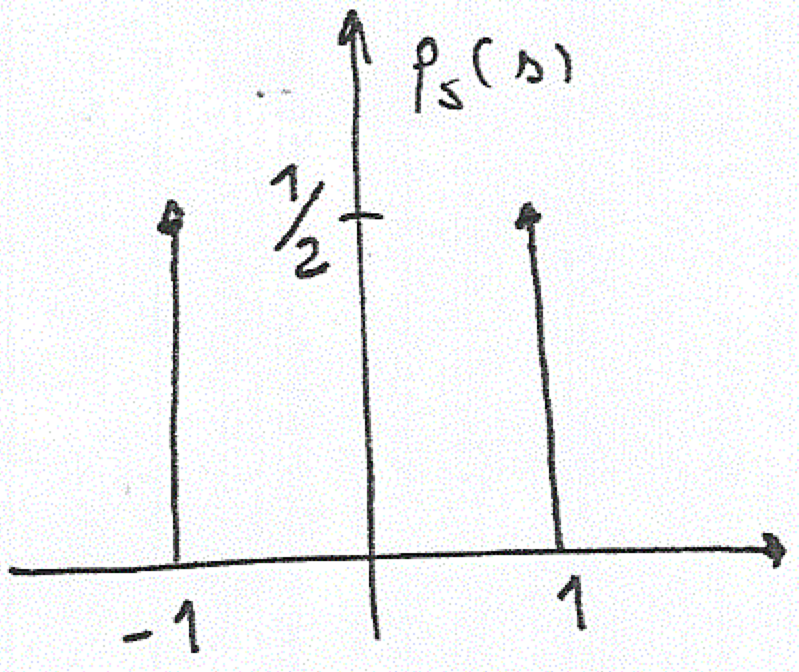
4

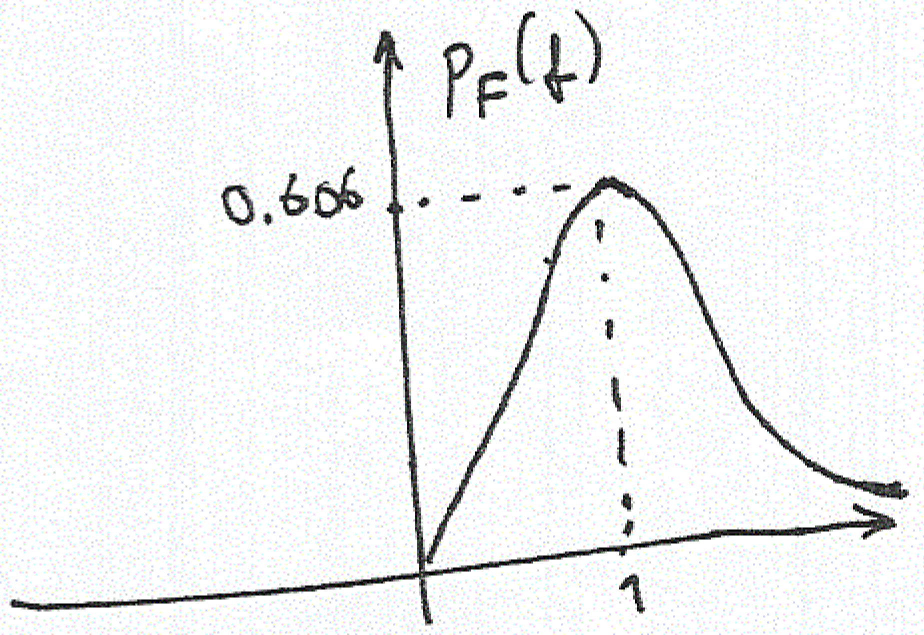
a)

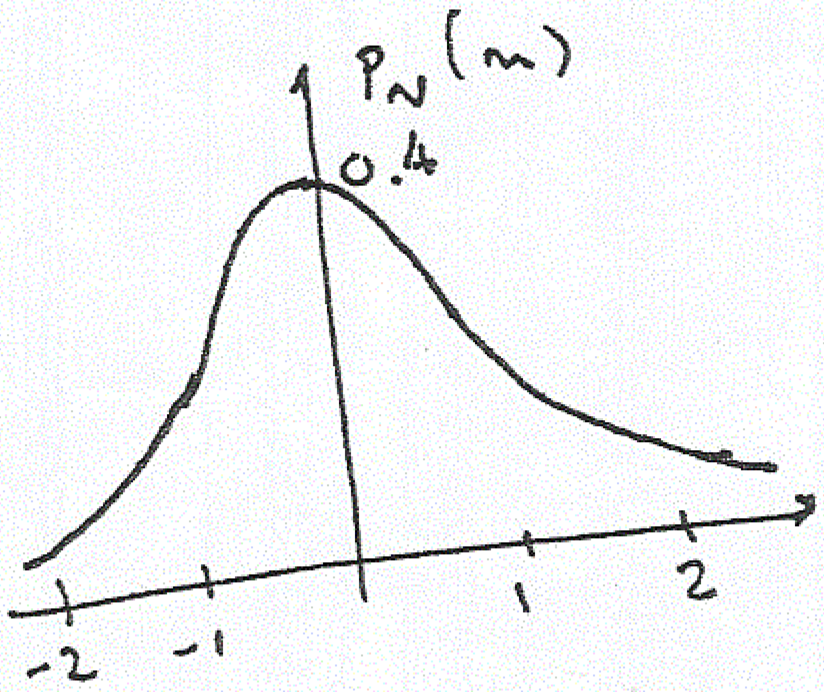
$$p_F(f) = f \cdot e^{-\frac{f^2}{2}}$$

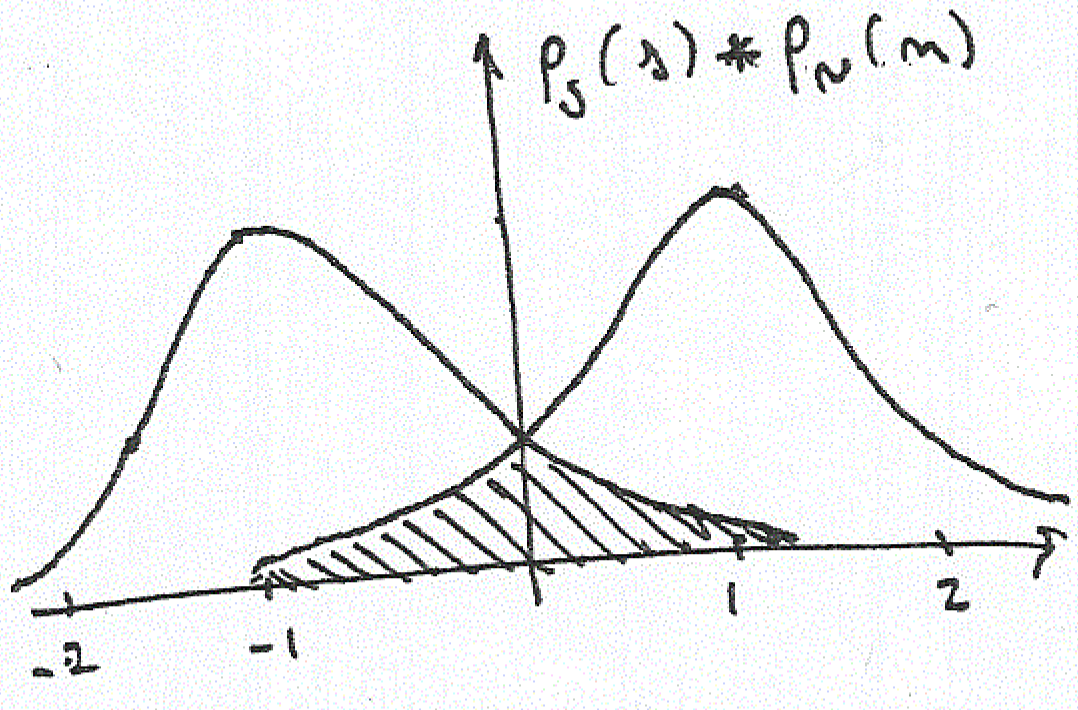
$$p_N(n) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{n^2}{2}}$$

$$p_S(s) = \frac{1}{2}\,\delta(s+1) + \frac{1}{2}\,\delta(s-1)$$









Com $m_N = 0$

$$P_{erro\,bit} = \left(\frac{1 - P(-2 \leq x \leq 0)}{2}\right) + \left(\frac{1 - P(0 \leq x \leq 2)}{2}\right) =$$

$$= \frac{0{,}317}{2} + \frac{0{,}317}{2} = 0{,}159 + 0{,}159 = 0{,}318 \simeq 31{,}8\%$$

$m_N = 1$

$$P_{erro\,bit} = (1 - P(x > 0)) + (1 - P(0 \leq x \leq 4)) =$$

$$= 0{,}5 + 0{,}0228 \simeq 0{,}5228 \simeq 52{,}28\%$$

Logo a Perrobit aumenta com o aumento da média de N.

b) A probabilidade de erro de bit permanece igual pois o ruído / distorção que o sinal recebe mantem-se, como tal a distorção dos bits não se altera independentemente se enviá mais ou menos bits a 1.