Math 403 Homework 8

Ben Pettipaw

Problem 14.2

The order of this factor group will be 4

Problem 14.6

The order of this factor group will be 36

Problem 14.10

The order of this element of the factor group will be 12

Problem 14.12

$< (1,1) >= \{(0,0)(1,1),(2,2),(3,3)\}$ thus $(3,1)+(3,1)=(2,2) \in< (1,1) >$
Therefore the order of the element is 2

Problem 14.16

Since $\rho_1^{-1} = \rho_2$ we have $\rho_1\mu_1\rho_1^{-1} = \rho_1\mu_1\rho_2 = \mu_1$
Also $\rho_1\rho_0\rho_1^{-1} = \rho_1\rho_0\rho_2 = \rho_0$
Therefore $i_{\rho_1}(H) = \{\rho_0, \mu_2\}$

Problem 14.23

(a) T **(b)** T
(c) T **(d)** T
(e) T **(f)** F
(g) T **(h)** F
(i) T **(j)** F

Problem 14.26

Because G is of finite order we know that the elements form an abelian subgroup T
Because G is abelian, every subgroup of G is normal so T is normal in G

Suppose xT is finite in G/T, thus $(xT)^m = T \implies x^m \in T$
Since G is a torsion group $(x^m)^r = x^{mr} = e$ thus x has finite order
Therefore $xT = T$ and the only finite element is the identity, thus G/T is torsion free

Problem 14.30

Since $m = (G : H) \implies |G/H| = m$ and since the order of each element divides the order of the group we have $(aH)^m = H$ for all $aH \in G/H$
Therefore we have that $a^m \in H \quad \forall a \in G$

Problem 14.34

Let $g \in G$
Because $i_g : G \to G$ is injective we have $|i_g(H)| = |H|$
Therefore H is invariant under all inner automorphisms of G, and is a normal subgroup of G