# Phys 227 Pset 1

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## Tests for Convergence

### 0.1 Preliminary Test:

If the terms of an infinite series do *not* tend to zero (that is, if $lim_{n\to\infty}a_n \neq 0$), the series diverges. If $lim_{n\to\infty}a_n = 0$, then this does not definitely diverge; however, we must test further.

### 0.2 Comparison Test:

To test a series $a$, take a series of positive terms $m$ we know converges. Then, if $|a_n| \leq m_n$, from some point $n$ and beyond, the series $a$ converges also.

#### 0.2.1 Example: test $\sum_{n=1}^{\infty}\frac{1}{n!}$for convergence

We have

$$\sum_{n=1}^{\infty}\frac{1}{n!} = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + ...$$

Choose the geometric series

$$\sum_{n=1}^{\infty}r^n$$

with $r = \frac{1}{2}$such that

$$\sum_{n=1}^{\infty}r^n = \sum_{n=1}^{\infty}(\frac{1}{2})^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$$

Then since $\sum_{n=1}^{\infty}\frac{1}{n!} < \sum_{n=1}^{\infty}(\frac{1}{2})^n$for $n \geq 3$, $\sum_{n=1}^{\infty}\frac{1}{n!}$ also converges.

### 0.3 Integral Test

For some series $a$, if the terms are of the series are positive and monotonically decrease, then if

$$\int^{\infty} a_n dn$$

converges, the series $a$ converges; if the integral diverges, the series diverges.

### 0.4 Ratio Test

For some series $a$, let $\rho_n$be the absolute value of the ratio of the terms $a_{n+1}/a_n$. In symbols,

$$\rho_n = |\frac{a_{n+1}}{a_n}|$$

and then the series $\rho$is

$$\rho = lim_{n\to\infty}\rho_n.$$

Then the ratio test states that $\rho < 1$, the series converges; if $\rho > 1$, the series diverges; if $\rho = 1$, this tells you nothing.

### 0.5 Special Comparison Test

Suppose we have two series with all terms larger than or equal to 0, $\sum a_n$and $\sum b_n$ . Then let $c = \lim_{n\to\infty} a_n/b_n$. If c is positive and finite, then either both series converge or both series diverge.