Recurrence analysis generalised

Given for example:

- $\mathrm{f}(x) = \lfloor x/2 \rfloor$
- $\mathrm{g}(x) = 2x + n$
- $\mathrm{h}(x) = x \lg x$
- $m = f(n)$

Consider this: (abstract conversion above, source example below)

1. The question takes the form: show $\mathrm{T}(n) = \mathrm{g}(\mathrm{T}(\mathrm{f}(n))) \in \mathrm{O}(\mathrm{h}(n))$.
   The question takes the form: show $\mathrm{T}(n) = 2\,\mathrm{T}(\lfloor n/2 \rfloor) + n \in \mathrm{O}(n \lg n)$.
2. Assume $\mathrm{T}(m) \leq c\,\mathrm{h}(m)$ when $m = \mathrm{f}(n)$.
   Assume $\mathrm{T}(m) \leq cm \lg m$ when $m = \lfloor n/2 \rfloor$.
3. Substitute $\mathrm{T}(m) \leq c\,\mathrm{h}(m)$ into $\mathrm{T}(n) = 2\,\mathrm{T}(\lfloor n/2 \rfloor) + n$.
   Substitute $\mathrm{T}(m) \leq cm \lg m$ into $\mathrm{T}(n) = 2\,\mathrm{T}(\lfloor n/2 \rfloor) + n$.
4. This yields $\mathrm{T}(n) \leq \mathrm{g}(c\,\mathrm{h}(\mathrm{f}(n)))$.
   This yields $\mathrm{T}(n) \leq 2c\lfloor n/2 \rfloor \lg \lfloor n/2 \rfloor + n$.
5. Use algebra to manipulate and reduce it to $\mathrm{T}(n) \leq c\,\mathrm{h}(n)$.
   Use algebra to manipulate and reduce it to $\mathrm{T}(n) \leq cn \lg n$.

$$\mathrm{T}(n) \leq 2c\lfloor n/2 \rfloor \lg \lfloor n/2 \rfloor + n \tag{1}$$
$$\mathrm{T}(n) \leq cn \lg \lfloor n/2 \rfloor + n \tag{2}$$
$$\mathrm{T}(n) \leq cn \lg n - cn \lg 2 + n \tag{3}$$
$$\mathrm{T}(n) \leq cn \lg n \tag{4}$$

6. Test and find the base case values n_0 and c.
   Test and find the base case values n_0 and c.