Engineering Probability and Statistics

Homework 12 Solutions
2 May 2014

Section 9.1: Hypothesis Testing

9-1 a) $H_0 : \mu = 25, H_1 : \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0 : \sigma > 10, H_1 : \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0 : \bar{x} = 50, H_1 : \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0 : p = 0.1, H_1 : p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0 : s = 30, H_1 : s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-2

The conclusion does not provide strong evidence that the critical dimension mean equals 100nm. There is not sufficient evidence to reject the null hypothesis.

9-10 a) α = P($\bar{X}$ ≤ 98.5) + P($\bar{X}$ > 101.5)

$$= P\left(\frac{\bar{X}-100}{2/\sqrt{9}} \leq \frac{98.5-100}{2/\sqrt{9}}\right) + P\left(\frac{\bar{X}-100}{2/\sqrt{9}} > \frac{101.5-100}{2/\sqrt{9}}\right)$$

= P(Z ≤ –2.25) + P(Z > 2.25)
= (P(Z ≤- 2.25)) + (1 – P(Z ≤ 2.25))
= 0.01222 + 1 – 0.98778
= 0.01222 + 0.01222 = 0.02444

9-11 Use n = 5, everything else held constant:

a) P($\bar{X}$ ≤ 98.5) + P($\bar{X}$ >101.5)

$$= P\left(\frac{\bar{X}-100}{2/\sqrt{5}} \leq \frac{98.5-100}{2/\sqrt{5}}\right) + P\left(\frac{\bar{X}-100}{2/\sqrt{5}} > \frac{101.5-100}{2/\sqrt{5}}\right)$$

= P(Z ≤ –1.68) + P(Z > 1.68)
= P(Z ≤ –1.68) + (1 – P(Z ≤ 1.68))
= 0.04648 + (1 – 0.95352) = 0.09296

9-12 $\mu_0 - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \bar{X} \leq \mu_0 + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$, where σ =2

a) α = 0.01, n = 9, then $z_{\alpha/2}$ =2,57, then 98.29,101.71

b) α = 0.05, n = 9, then $z_{\alpha/2}$ =1.96, then 98.69,101.31

c) α = 0.01, n = 5, then $z_{\alpha/2}$ = 2.57, then 97.70,102.30

d) $\alpha = 0.05$, n = 5, then $z_{\alpha/2}$ =1.96, then $98.25, 101.75$

9-14 a) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi\left(\left|\frac{98-100}{2/\sqrt{9}}\right|\right)$) = 2(1- $\Phi(3)$) = 2(1-0.99865) = 0.0027

b) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi\left(\left|\frac{101-100}{2/\sqrt{9}}\right|\right)$) = 2(1- $\Phi(1.5)$) = 2(1-0.93319) = 0.13362

c) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi\left(\left|\frac{102-100}{2/\sqrt{9}}\right|\right)$) = 2(1- $\Phi(3)$) = 2(1-0.99865) = 0.0027

Section 9.2: Tests on the Mean of a Normal Distribution, Variance Known

9-33(9-35)

a) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi(2.05)$) $\cong$ 0.04

b) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi(1.84)$) $\cong$ 0.066

c) P-value = 2(1- $\Phi(|Z_0|)$) = 2(1- $\Phi(0.4)$) $\cong$ 0.69

9-41(9-43)

a) 1) The parameter of interest is the true mean crankshaft wear, μ.
2) $H_0 : \mu = 3$
3) $H_1 : \mu \neq 3$
4) $z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.05$ and $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.05$ and $z_{0.025} = 1.96$
6) $\bar{x} = 2.78$, $\sigma = 0.9$

$$z_0 = \frac{2.78-3}{0.9/\sqrt{15}} = -0.95$$

7) Because –0.95 > -1.96 fail to reject the null hypothesis. There is not sufficient evidence to support the claim the mean crankshaft wear differs from 3 at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.025} + \frac{3-3.25}{0.9/\sqrt{15}}\right) - \Phi\left(-z_{0.025} + \frac{3-3.25}{0.9/\sqrt{15}}\right)$

= $\Phi(1.96 + -1.08) - \Phi(-1.96 + -1.08)$
= $\Phi(0.88) - \Phi(-3.04)$
= 0.81057 – (0.00118)
= 0.80939

c) $$n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.025} + z_{0.10})^2 \sigma^2}{(3.75-3)^2} = \frac{(1.96+1.29)^2 (0.9)^2}{(0.75)^2} = 15.21, \quad n \cong 16$$

Section 9.3: Tests on the Mean of a Normal Distribution, Variance Unknown

9-49(9-53)

a) $\alpha = 0.01$, n = 20, the critical value = 2.539
b) $\alpha = 0.05$, n = 12, the critical value = 1.796
c) $\alpha = 0.1$, n = 15, the critical value = 1.345

9-60(9-64)

a) 1) The parameter of interest is the true mean rainfall, μ.

2) $H_0 : \mu = 25$

3) $H_1 : \mu > 25$

4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

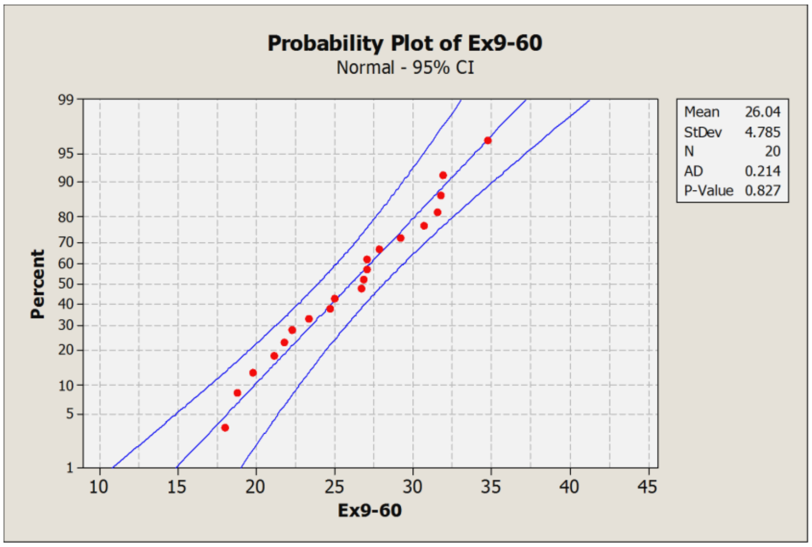
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.01$ and $t_{0.01,19} = 2.539$ for $n = 20$

6) $\bar{x} = 26.04$ $s = 4.78$ $n = 20$

$$t_0 = \frac{26.04 - 25}{4.78/\sqrt{20}} = 0.97$$

7) Because 0.97 < 2.539 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean rainfall is greater than 25 acre-feet at $\alpha = 0.01$. The $0.10 < \text{P-value} < 0.25$.

b) The data on the normal probability plot falls along a line. Therefore, the normality assumption is reasonable.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27 - 25|}{4.78} = 0.42$

Using the OC curve, Chart VII h) for $\alpha = 0.01$, $d = 0.42$, and $n = 20$, obtain $\beta \cong 0.7$ and power of $1 - 0.7 = 0.3$.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27.5 - 25|}{4.78} = 0.52$

Using the OC curve, Chart VII h) for $\alpha = 0.05$, $d = 0.42$, and $\beta \cong 0.1$ (Power=0.9), $n = 75$

e) 99% lower confidence bound on the mean diameter

$$\bar{x} - t_{0.01,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$26.04 - 2.539\left(\frac{4.78}{\sqrt{20}}\right) \le \mu$$

$$23.326 \le \mu$$

Because the lower limit of the CI is less than 25 there is insufficient evidence to conclude that the true mean rainfall is greater than 25 acre-feet at $\alpha = 0.01$.

Section 9.4: Tests on Variance of Normal Distribution

9-71(9-77)

a) $\alpha = 0.01$, n = 20, from Table V we find the following critical values 6.84 and 38.58
b) $\alpha = 0.05$, n = 12, from Table V we find the following critical values 3.82 and 21.92
c) $\alpha = 0.10$, n = 15, from Table V we find the following critical values 6.57 and 23.68

9-79(9-85)

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the solution can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = (0.25)^2$
3) $H_1: \sigma^2 \neq (0.25)^2$
4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.995,50} = 32.36$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.005,50} = 71.42$ for n = 51
6) n = 51, s = 0.37

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.37)^2}{(0.25)^2} = 109.52$$

7) Because 109.52 > 71.42 reject H_0. The standard deviation of titanium percentage is significantly different from 0.25 at $\alpha = 0.01$. P-value/2 < 0.005, then P-value < 0.01

b) 95% confidence interval for σ:
First find the confidence interval for σ^2:
For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{71.42} \le \sigma^2 \le \frac{50(0.37)^2}{32.36}$$

$$0.096 \le \sigma^2 \le 0.2115$$

Taking the square root of the endpoints of this interval we obtain, $0.31 < \sigma < 0.46$

Because 0.25 falls below the lower confidence bound we conclude that the population standard deviation is not equal to 0.25.

Section 11-2

11-3(11-5)

a) **Regression Analysis: Rating Pts versus Yds per Att**

```
The regression equation is
Rating Pts = 14.2 + 10.1 Yds per Att

Predictor      Coef  SE Coef     T      P
Constant     14.195    9.059  1.57  0.128
Yds per Att  10.092    1.288  7.84  0.000

S = 5.21874   R-Sq = 67.2%   R-Sq(adj) = 66.1%

Analysis of Variance
Source          DF      SS      MS      F      P
Regression       1  1672.5  1672.5  61.41  0.000
Residual Error  30   817.1    27.2
Total           31  2489.5
```

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$S_{xx} = 1583.442 - \frac{(223.93)^2}{32} = 16.422$$

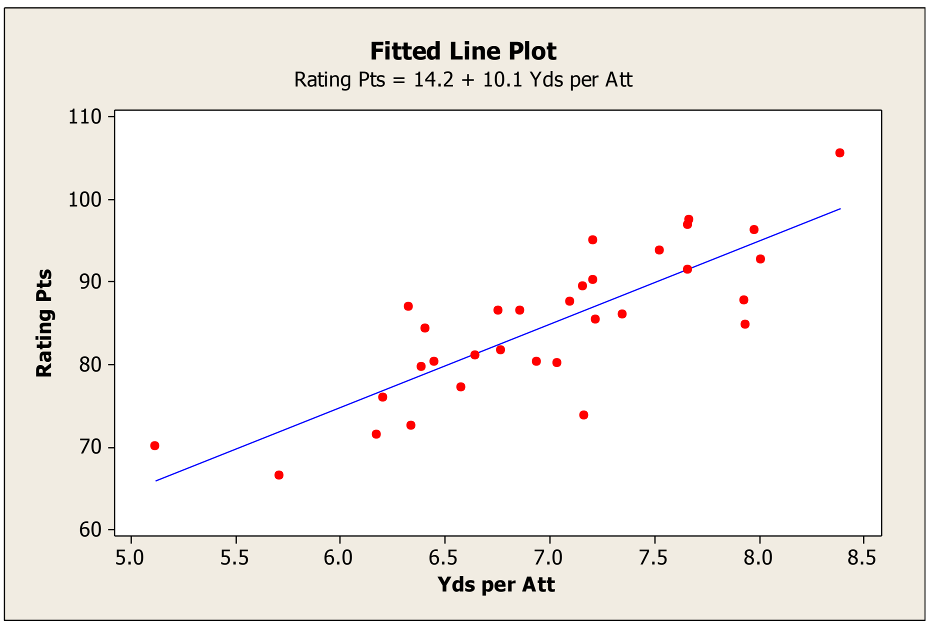
$$S_{xy} = 19158.49 - \frac{(223.93)(2714.1)}{32} = 165.7271$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{165.7271}{16.422} = 10.092$$

$$\hat{\beta}_0 = \frac{2714.1}{32} - (10.092)\left(\frac{223.93}{32}\right) = 14.195$$

$$\hat{y} = 14.2 + 10.1x$$

$$\hat{\sigma}^2 = MS_E = \frac{SS_E}{n-2} = \frac{817.1}{30} = 27.24$$



b) $\hat{y} = 14.2 + 10.1(7.5) = 89.95$

c) $-\hat{\beta}_1 = -10.1$

d) $\frac{1}{10.1} \times 10 = 0.99$

e) $\hat{y} = 14.2 + 10.1(7.21) = 87.02$

There are two residuals

$$e = y - \hat{y}$$

$$e_1 = 90.2 - 87.02 = 3.18$$

$$e_2 = 95 - 87.02 = 7.98$$

11-5(11-7)

a)

```
Regression Analysis - Linear model: Y = a+bX
Dependent variable: Usage              Independent variable: Temperature
--------------------------------------------------------------------------------
                               Standard            T            Prob.
Parameter        Estimate        Error           Value          Level
Intercept         -6.3355       1.66765        -3.79906         .00349
Slope             9.20836      0.0337744        272.643         .00000
--------------------------------------------------------------------------------
                           Analysis of Variance
Source              Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                    280583.12      1    280583.12    74334.4       .00000
Residual                 37.746089     10     3.774609
--------------------------------------------------------------------------------
Total (Corr.)            280620.87     11
Correlation Coefficient = 0.999933          R-squared =  99.99 percent
Stnd. Error of Est. = 1.94284
```

$$\hat{\sigma}^2 = 3.7746$$

If the calculations were to be done by hand, use Equations (11-7) and (11-8).

$$\hat{y} = -6.3355 + 9.20836x$$

b) $\hat{y} = -6.3355 + 9.20836(55) = 500.124$

c) If monthly temperature increases by $1^{\circ}\mathrm{F}$, $\hat{y}$ increases by 9.208

d) $\hat{y} = -6.3355 + 9.20836(47) = 426.458$

$\hat{y} = 426.458$

$$e = y - \hat{y} = 424.84 - 426.458 = -1.618$$

11-6(11-8)

a)

```
The regression equation is
MPG = 39.2 - 0.0402 Engine Displacement


Predictor                 Coef   SE Coef      T      P
Constant                39.156     2.006  19.52  0.000
Engine Displacement  -0.040216  0.007671  -5.24  0.000


S = 3.74332   R-Sq = 59.1%   R-Sq(adj) = 57.0%


Analysis of Variance

Source          DF      SS      MS      F      P
Regression       1  385.18  385.18  27.49  0.000
Residual Error  19  266.24   14.01
Total           20  651.41
```

$$\hat{\sigma}^2 = 14.01$$

$$\hat{y} = 39.2 - 0.0402x$$

b) $\hat{y} = 39.2 - 0.0402(150) = 33.17$

c) $\hat{y} = 34.2956$

$$e = y - \hat{y} = 41.3 - 34.2956 = 7.0044$$

Section 11-4

11-25(11-31)

a) **Regression Analysis: Rating Pts versus Yds per Att**

```
The regression equation is
Rating Pts = 14.2 + 10.1 Yds per Att

Predictor      Coef  SE Coef     T      P
Constant     14.195    9.059  1.57  0.128
Yds per Att  10.092    1.288  7.84  0.000

S = 5.21874   R-Sq = 67.2%   R-Sq(adj) = 66.1%

Analysis of Variance
Source          DF      SS      MS      F      P
Regression       1  1672.5  1672.5  61.41  0.000
Residual Error  30   817.1    27.2
Total           31  2489.5
```

Refer to the ANOVA

$$H_0 : \beta_1 = 0$$
$$H_1 : \beta_1 \neq 0$$
$$\alpha = 0.01$$

Because the P-value = 0.000 < α = 0.01, reject H_0. If the assumptions are valid, we conclude that there is a useful linear relationship between these two variables.

b) $\hat{\sigma}^2 = 27.2$

$$se(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} = \sqrt{\frac{27.2}{16.422}} = 1.287$$

$$se(\hat{\beta}_0) = \sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right]} = \sqrt{27.2\left[\frac{1}{32} + \frac{7^2}{16.422}\right]} = 9.056$$

c) 1)The parameter of interest is the regressor variable coefficient β_1.

2) $H_0: \beta_1 = 10$

3) $H_1: \beta_1 \neq 10$

4) $\alpha = 0.01$

5) The test statistic is $t_0 = \dfrac{\hat{\beta}_1 - \beta_{1,0}}{se(\hat{\beta}_1)}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-2}$ where $-t_{0.005,30} = -2.750$ or $t_0 > t_{0.005,30} = 2.750$

7) Using the results from Exercise 10-6

$$t_0 = \frac{10.092 - 10}{1.287} = 0.0715$$

8) Because 0.0715 < 2.750, fail to reject H_0. There is not enough evidence to conclude that the slope differs from 10 at α = 0.01.

11-27(11-33)

Refer to the ANOVA for Exercise 10-65

a) 1) The parameter of interest is the regressor variable coefficient, β_1.

2) $H_0:\beta_1 = 0$

3) $H_1:\beta_1 \neq 0$

4) $\alpha = 0.01$

5) The test statistic is $f_0 = \dfrac{MS_R}{MS_E} = \dfrac{SS_R\ /1}{SS_E\ /(n-2)}$

6) Reject H_0 if $f_0 > f_{\alpha,1,22}$ where $f_{0.01,1,10} = 10.049$

7) Using the results from Exercise 10-6

$$f_0 = \frac{280583.12/1}{37.746089/10} = 74334.4$$

8) Since 74334.4 > 10.049, reject H_0 and conclude the model is useful $\alpha = 0.01$. P-value < 0.000001

b) $se(\hat{\beta}_1) = 0.0337744$, $se(\hat{\beta}_0) = 1.66765$

c) 1) The parameter of interest is the regressor variable coefficient, β_1.

2) $H_0:\beta_1 = 10$

3) $H_1:\beta_1 \neq 10$

4) $\alpha = 0.01$

5) The test statistic is $t_0 = \dfrac{\hat{\beta}_1 - \beta_{1,0}}{se(\hat{\beta}_1)}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-2}$ where $-t_{0.005,10} = -3.17$ or $t_0 > t_{0.005,10} = 3.17$

7) Using the results from Exercise 10-6

$$t_0 = \frac{9.21-10}{0.0338} = -23.37$$

8) Since $-23.37 < -3.17$ reject H_0 and conclude the slope is not 10 at $\alpha = 0.01$. P-value $\approx$ 0.

d) H_0: $\beta_0 = 0$ H_1: $\beta_0 \neq 0$

$$t_0 = \frac{-6.3355-0}{1.66765} = -3.8$$

P-value < 0.005. Reject H_0 and conclude that the intercept should be included in the model.

11-28(11-34)

Refer to the ANOVA for Exercise 11-6

$$H_0 : \beta_1 = 0; H_1 : \beta_1 \neq 0$$

(a) $f_0 = \dfrac{MS_R}{MS_E} = \dfrac{385.18}{14.01} = 27.49$

$$F_{0.01,1,19} = 8.18$$

Reject the null hypothesis and conclude that the slope is not zero. The exact *P*-value is *P* = 0.0000463

(b) From the computer output from Exercise 11-6:

$se(\hat{\beta}_0) = 2.006, \quad se(\hat{\beta}_1) = 0.007671$

(c)

$H_0 : \beta_1 = -0.05; H_1 : \beta_1 < -0.05$

$$t_0 = \frac{\hat{\beta}_1 - \hat{\beta}_{1,0}}{se(\hat{\beta}_1)} = \frac{-0.040216 - (-0.05)}{0.007671} = \frac{0.090216}{0.007671} = 11.76$$

$t_{0.01,19} = 2.539,$ since t_0 is not less than -$t_{0.01,19} = -2.539,$ do not reject H_0

$P \cong 1.0$

(d)

$H_0 : \beta_0 = 0; H_1 : \beta_0 \neq 0$

$$t_0 = \frac{\hat{\beta}_0 - \hat{\beta}_{0,0}}{se(\hat{\beta}_0)} = \frac{39.156 - 0}{2.006} = 19.52$$

$t_{0.005,19} = 2.861,$ since $|t_0| > t_{0.005,19}$ reject H_0

$P = 4.95E - 14 \cong 0$

Sections 11-5 and 11-6

11-41(11-49)

$t_{\alpha/2,n-2} = t_{0.025,30} = 2.042$

a) 95% confidence interval on β_1

$\hat{\beta}_1 \pm t_{\alpha/2,n-2} se(\hat{\beta}_1)$

$10.092 \pm t_{0.025,30}(1.287)$

$10.092 \pm 2.042(1.287)$

$7.464 \leq \beta_1 \leq 12.720$

b) 95% confidence interval on β_0

$\hat{\beta}_0 \pm t_{\alpha/2,n-2} se(\hat{\beta}_0)$

$14.195 \pm t_{0.025,30}(9.056)$

$14.195 \pm 2.042(9.056)$

$-4.297 \leq \hat{\beta}_0 \leq 32.687$

c) 95% confidence interval for the mean rating when the average yards per attempt is 8.0

$\hat{\mu} = 14.195 + 10.092(8.0) = 94.931$

$$\hat{\mu} \pm t_{0.025,30} \sqrt{\hat{\sigma}^2 \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right)}$$

$$94.931 \pm 2.042 \sqrt{27.2 \left(\frac{1}{32} + \frac{(8-7)^2}{16.422} \right)}$$

$91.698 \leq \mu \leq 98.164$

d) 95% prediction interval on $x_0 = 8.0$

$$\hat{y} \pm t_{0.025,30}\sqrt{\hat{\sigma}^2\left(1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}\right)}$$

$$94.931 \pm 2.042\sqrt{27.2\left(1+\frac{1}{32}+\frac{(8-7)^2}{16.422}\right)}$$

$$83.801 \le \mu \le 106.061$$

11-43(11-51)

Regression Analysis: Usage versus Temperature

```
The regression equation is
Usage = - 6.34 + 9.21 Temperature


Predictor        Coef  SE Coef       T      P
Constant       -6.336    1.668   -3.80  0.003
Temperature   9.20836  0.03377  272.64  0.000


S = 1.94284   R-Sq = 100.0%   R-Sq(adj) = 100.0%


Analysis of Variance

Source          DF      SS      MS         F      P
Regression       1  280583  280583  74334.36  0.000
Residual Error  10      38       4
Total           11  280621
```

a) 9.20836 – 2.228(0.03377) = 9.101 ≤ β_1 ≤ 9.20836 + 2.228(0.03377) = 9.932
b) -6.33550 – 2.228(1.66765) = –11.622 ≤ β_0 ≤ -6.33550 + 2.228(1.66765) = –1.050

c) $500.124 \pm (2.228)\sqrt{3.774609(\frac{1}{12}+\frac{(55-46.5)^2}{3308.9994})}$

500.124 ± 1.4037586

$498.72024 \le \hat{\mu}_{Y|x_0} \le 501.52776$

d) $500.124 \pm (2.228)\sqrt{3.774609(1+\frac{1}{12}+\frac{(55-46.5)^2}{3308.9994})}$

500.124 ± 4.5505644

$495.57344 \le y_0 \le 504.67456$

It is wider because the prediction interval includes errors for both the fitted model and for a future observation.

11-44(11-52)

Refer to the ANOVA for Exercise 11-6.

(a) $t_{0.025,\,19} = 2.093$

$$34.96 \le \beta_0 \le 43.36; -0.0563 \le \beta_1 \le -0.0241$$

(b)

Descriptive Statistics: x = displacement

Variable	n	Mean	Sum	Sum of Squares
x	21	238.9	5017.0	1436737.0

$\hat{y} = 33.15$ when $x = 150$

$$33.15 \pm 2.093\sqrt{14.01\left[\frac{1}{21} + \frac{(150 - 238.9)^2}{1,436,737.0}\right]}$$

33.15 ± 1.8056

$31.34 \le \mu_{Y|x=150} \le 34.96$

(c)

$\hat{y} = 33.15$ when $x = 150$

$$33.15 \pm 2.093\sqrt{14.01\left[1 + \frac{1}{21} + \frac{(150 - 238.9)^2}{1,436,737.0}\right]}$$

33.15 ± 8.0394

$25.11 \le Y_0 \le 41.19$