CS 331 Assignment #6

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Exercise 1

We know by $gap(p, i, j) = gap(p, j, k) = 0$ that $s(i) * g(i, j) \leq f(p, i, j)$ and $s(j) * g(j, k) \leq f(p, j, k)$. With this knowledge, we must prove $gap(p, i, k) = 0$, meaning $s(i) * g(i, k) \leq f(p, i, k)$. We will first examine the case where $i < j < k$.

Notice that since $\beta(i).quality < \beta(j).quality < \beta(k).quality$, $g(i, j)$, $g(j, k)$ and $g(i, k)$ are all non-positive.

Suppose that $s(i) * g(i, k) > f(p, i, k)$. Equivalently, $s(i) * g(i, k) > p_{\beta(k)} - p_{\beta(i)}$. However, by the other two formulas, we may state that this is unequivocally false: the fact that $f(p, i, j) \geq s(i) * g(i, j)$ and $f(p, j, k) \geq s(j) * g(j, k)$ sufficiently restrict $f(p, i, k)$ by virtue of $g(i, j) \leq 0$, $g(j, k) \leq 0$, and $s(i) < s(j)$ such that it is less than $s(i) * g(i, k)$.

The other direction where $k < j < i$ plays out similarly, but since $\beta(k).quality < \beta(j).quality < \beta(i).quality$, all the relevant applications of function g will now be non-negative. In addition, now $s(k) < s(j) < s(i)$, making $s(i)$ the largest slope and thus restricting $f(p, i, k)$ to be greater than or equal to $s(i) * g(i, k)$.

Exercise 2

Choose a k such that $\alpha(k)$ is linear and there exists one distinct $\alpha(k')$ such that $k' < k$ and $\alpha(k')$ is linear. Choose $j = \sigma(k)$. Then, $gap(p, j, k) = 0$ because $L(p, n)$ holds. Choose $i = \tau(j)$. Then, $gap(p, i, j) = 0$ because $R(p, 0)$ holds. By Exercise 1, $gap(p, i, k) = 0$. By induction on k, we may increase k until $k = j$ such that $\forall i, j \in [n], gap(p, i, j) = 0$.